Stability of A MIMO Reset Control System Under Constant Inputs [1]

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Abstract

Reset controllers are standard linear compensators equipped with mechanism to instantaneously reset their states. With respect to pure linear control, there is evidence that this reset action is capable of improving control system tradeoffs. Recently, in [1] we established stability conditions for SISO reset control systems and this paper extends these results to the MIMO case. The paper's objective is to analyze the stability of such reset control systems when excited by constant inputs. Our main result gives conditions under which the equilibrium point of the closed-loop dynamic is asymptotically stable.

1 Introduction

The MIMO reset control system we consider is shown in Figure 1 where the first-order reset element (FORE) is a diagonal compensator described by the resetting differential equation

$$\begin{aligned} \dot{x}_c &= -bx_c + e; \quad e \neq 0 \\ x_c &= 0; \quad e = 0 \\ u &= x_c; \end{aligned} \tag{1}$$

where $x_c \in \mathbb{R}^n$ is the controller state, u is the reset element's output and $b > 0$ is the FORE's pole for each diagonal entry. In the absence of resetting the

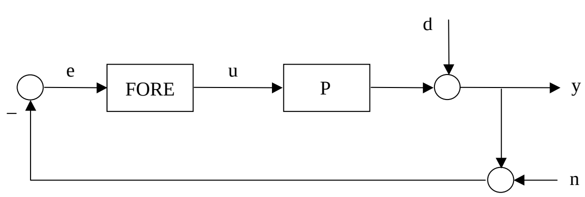

Figure 1: Block Diagram of a MIMO Reset System

FORE behaves as the linear system $\frac{1}{s+b}I$. We take the MIMO plant transfer matrix as

$$P(s) = \frac{s+b}{s}K(Ms+D)^{-1}$$

where K, M and $D \in \mathbb{R}^{n\times n}$. The plant $P(s)$ is purposely chosen so that the associated linear complementary sensitivity function has a standard MIMO second order form

$$T(s) = K(Ms^2 + Ds + K)^{-1}.$$

$T(s)$ can be thought of as the transfer function for a generalized mechanical system where M denotes the inertia matrix, D the damping matrix and K the spring matrix.

Assumption 1: Matrices M, D and K are symmetric, positive-definite and mutually commutable. Furthermore, the reset pole b is chosen such that $(bM - D)$ is positive definite.

Under this assumption $T(s)$ is stable.

2 Stability Conditions for the Closed-Loop Resetting Differential Equation

In this section we model the reset control system by a resetting differential equation and study its asymptotic stability. A state space representation for $P(s)$ is

$$\begin{aligned} \dot{x}_p &= Ax_p + Bu \\ y &= Cx_p + d \end{aligned}$$

where $x_p \in \mathbb{R}^{2n}$ is the plant state and

$$\begin{aligned} A &= \begin{bmatrix} -M^{-1}D & M^{-1} \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} M^{-1} \\ bI \end{bmatrix}; \\ C &= \begin{bmatrix} K & 0 \end{bmatrix}. \end{aligned}$$

For simplicity we focus on the response to constant disturbances and take $r = 0$, $n = 0$ and $d(t) \equiv d_0$. Together with (1), the control system can then be described by the resetting differential equation

$$\dot{x} = A_{c\ell}x - \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} d_0; \quad x \notin N_{d_0} \tag{2}$$

[1] This material is based upon work supported by the National Science Foundation under Grant No. CMS-9800612.

$$\Delta x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -I \end{bmatrix} x; \quad x \in N_{d_0}$$

where $x = \begin{bmatrix} x_p' & x_c' \end{bmatrix}'$ is the closed-loop state,

$$A_{c\ell} = \begin{bmatrix} -M^{-1}D & M^{-1} & M^{-1} \\ 0 & 0 & bI \\ -K & 0 & -bI \end{bmatrix}$$

and N_{d_0} is the *set of reset states*

$$N_{d_0} = \{\xi : C_y \xi = -d_0; x_c \neq 0\}$$

where

$$C_y = \begin{bmatrix} K & 0 & 0 \end{bmatrix}.$$

For a given initial condition $x_0 \triangleq x(0)$, let $\phi(x_0, t)$ denote the unique solution to (2). Then, the *set of reset times* $T(x_0)$ is defined as

$$T(x_0) \triangleq \{t > 0 : C_y \phi(x_0, t) = -d_0; x_c \neq 0\}.$$

We make the following assumption on this set.

Assumption 2: Given initial condition x_0, the associated set of reset times $T(x_0)$ is an unbounded discrete subset of $\mathbb{R}^+$.

The equilibrium state x_e for (2) is

$$x_e = A_{c\ell}^{-1} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} d_0 = -K^{-1} \begin{bmatrix} D \\ I \\ 0 \end{bmatrix} d_0.$$

To establish its asymptotic stability, we consider the quadratic Lyapunov candidate $V(x) = x'Px$ and state a special case of the general stability result in Theorems 13.1 and 13.2 of [2].

Theorem 1: (see [5] for proof) *Under Assumption 2, the equilibrium state x_e is asymptotically stable if there exists a positive-definite symmetric matrix P such that*

$$x' \left(A_{c\ell}' P + P A_{c\ell}\right) x < 0; \quad x \notin N \tag{3}$$

and

$$x' \left(A_R' P A_R - P\right) x \leq 0; \quad x \in N \tag{4}$$

where

$$A_R \triangleq \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$N \triangleq \{\xi : C_y \xi = 0\}.$$

□

3 Stability of The Reset Control System

In this section we show that the reset control system described in (2) is asymptotically stable. We do this by using Theorem 1 and proving that there exists a positive-definite symmetric matrix P such that (3) and (4) hold.

Theorem 2: (see [5] for proof) *Under Assumptions 1 and 2, there exists a positive-definite matrix P such that* (3) *and* (4) *hold, consequently the equilibrium state x_e is asymptotically stable.* □

4 Conclusions

In this paper we have shown that the MIMO reset control system in Figure 1 is asymptotically stable when either r, d or n is a constant. Since the associated linear system $T(s)$ also enjoys this same property, then the real benefit of our work comes in combining it with performance studies such as in [6] and [7] which demonstrated the potential of reset control to improve tradeoffs between competing control objectives.

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