Plant with Integrator: An Example of Reset Control Overcoming Limitations of Linear Feedback*

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Abstract

The purpose of this paper is twofold. First, to give conditions under which linear feedback control of a plant containing integrator must overshoot. Secondly, to give an example of reset control that does not overshoot under such constraints.

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1 Introduction

Reset control was introduced by Clegg [1] as a means to overcome the limitations of linear feedback. Roughly speaking, a reset element is a linear control element whose state is “reset” to zero when its input equals zero. Clegg applied this concept to an integrator element (the Clegg integrator) where the rationale for improved feedback performance came from this element’s favorable describing function. Krishnan and Horowitz [2] advanced this idea by resetting a first-order lag filter (the so-called first-order reset element (FORE)) which in [3] was incorporated in a quantitative design procedure. Recently, there has been renewed attention to this topic from both a theoretic and applications viewpoint; e.g., see [4] – [11]. One missing element in this work is a concrete example showing that reset control meets control system specifications that are unattainable over all linear controllers. This paper provides such an example. Switching control, a nonlinear scheme analogous to reset, exhibits similar advantage as claimed in [12]. The paper is organized as follows. In the next section we introduce an overshoot limitation on linear feedback systems arising when the loop contains an integrator. While new, this result is an immediate consequence of the time-domain limitations introduced in [13]. In Section 3 we give an example showing that reset control does not suffer this limitation. In Section 4 we conclude.

2 Overshoot in a linear feedback system containing integrator

Consider the standard linear feedback control system in Figure 1 where the plant $P(s)$ contains an integrator. Assume that $C(s)$ stabilizes. In [13] it was shown that the tracking

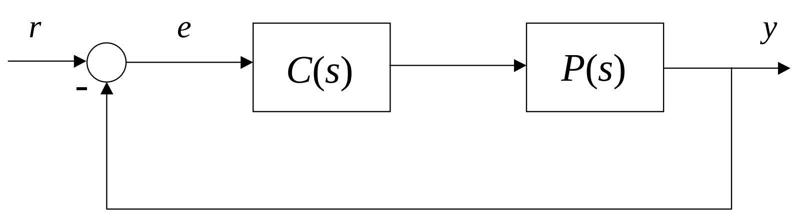

Figure 1: Linear feedback control system.

error e due to a unit-step input satisfies

$$\int_0^\infty e(t)dt = \frac{1}{K_v}$$

where the velocity constant K_v is defined by $K_v \triangleq lim_{s\to 0} sP(s)C(s)$. Alone, this constraint does not imply overshoot in the step response y; i.e., $y(t) \geq 1$ for some $t > 0$. However, introduction of an additional, sufficiently stringent time-domain bandwidth constraint will.

To see this, consider the notion of rise time t_r introduced in [13]:

$$t_r = \sup_T \left\{ T : y(t) \leq \frac{t}{T},\ t \in [0, T] \right\}.$$

The following result is immediate.

Proposition: *If $t_r > \frac{2}{K_v}$; i.e., the rise time is sufficiently slow, the unit-step response $y(t)$ overshoots.*

Proof: Clearly,

$$\begin{aligned} \frac{1}{K_v} &= \int_0^\infty e(t)dt \\ &\geq \int_0^{t_r} \left(1 - \frac{t}{t_r}\right) dt + \int_{t_r}^\infty e(t)dt \\ &= \frac{t_r}{2} + \int_{t_r}^\infty e(t)dt. \end{aligned}$$

Thus,

$$\int_{t_r}^\infty e(t)dt \leq \frac{1}{K_v} - \frac{t_r}{2}.$$

Since $t_r > \frac{2}{K_v}$, then $e(t) < 0$ (and hence $y(t) > 1$) for some $t \in (t_r, \infty)$. $\square$

Example: Consider the linear feedback system in Figure 1 where the plant $P(s)$ is simply an integrator. In addition to closed-loop stability suppose the design objectives are the following:

(i) Steady-state error no greater than one when tracking a unit-ramp input.

(ii) Rise time greater than 2 seconds when tracking a unit-step.

(iii) No overshoot in the step response.

To meet the error specification on the ramp response, this linear feedback system must have velocity error constant $K_v \geq 1$. Since $t_r > 2 \geq \frac{2}{K_v}$, the Proposition indicates that no stabilizing $C(s)$ exists to meet all the above objectives. In the next section we show that a reset control system can meet these specifications.

3 Reset control meets example's specifications

We reconsider the previous example using reset control (see Figure 2) and use a first-order reset element (FORE), see [3], described by

$$\begin{aligned} \dot{u}(t) &= -u(t) + e(t), \quad & e(t) \neq 0; \\ u(t^+) &= 0, & e(t) = 0. \end{aligned}$$

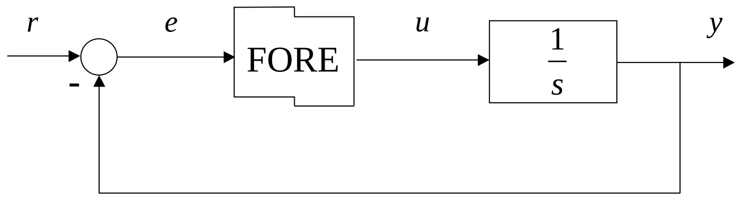

Figure 2: Reset control of an integrator using a first-order reset element.

In the following we let *base-linear* system refer to the reset control system in the absence of resetting. To address the preceding design objectives we first compute the tracking error of this base-linear system to a unit-ramp:

$$e_{ramp}(t) = 1 + \frac{2\sqrt{3}}{3} e^{-0.5t} \sin\left(\frac{\sqrt{3}}{2} t - \frac{\pi}{3}\right).$$

Since $e_{ramp}(t) \neq 0$, the reset control system never resets and its response is equivalent to the base-linear response. $K_v = 1$ for the base-linear system guarantees a steady-state error less than 1. The error of the base-linear control system to a unit-step input is

$$e_{step}(t) = \frac{2\sqrt{3}}{3} e^{-0.5t} \left(\sin\left(\frac{\sqrt{3}}{2} t\right) - \sin\left(\frac{\sqrt{3}}{2} t - \frac{\pi}{3}\right)\right).$$

This error first goes to zero at $t = \frac{4\sqrt{3}}{9}\pi \approx 2.42$. Thus, the first reset time is approximately 2.42 seconds. The error of the reset control system remains zero therafter since its response is deadbeat. This occurs since $(u, y) = (0, r)$ is an equilibrium point. The rise time t_r is determined from the tangency of $y(t) = 1 - e_{step}(t)$ with $\bar{y}(t) \triangleq mt$. Indeed, if $\frac{1}{m} < 2.42$ and these curves are tangent, then $t_r = \frac{1}{m}$. We thus seek a pair (t, m) satisfying

$$y(t) = \bar{y}(t)$$

and

$$\dot{y}(t) = \dot{\bar{y}}(t).$$

This occurs for $m = \frac{1}{2.35}$. Since $2.35 < 2.42$ we conclude $t_r = 2.35$ secs. This reset control system thus satisfies the second design constraint. The overshoot objective is met since the step response is deadbeat. Consequently, this reset control system meets objectives not attainable using linear feedback control. We confirm this analysis with simulation results shown in Figures 3 and 4. Finally, one can show the unforced reset control system to be asymptotically stable and to track step inputs with zero steady-state error; see [7] and [9] for more details.

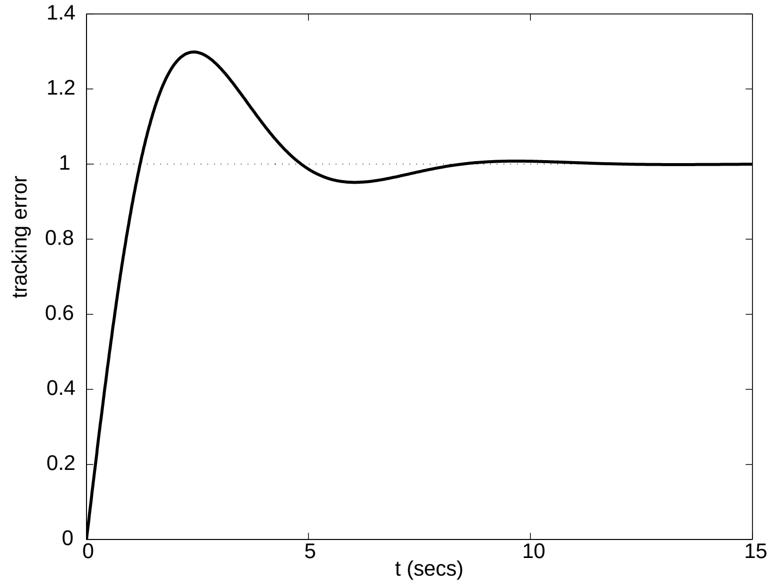

Figure 3: Tracking-error e to a unit-ramp input for the reset control system.

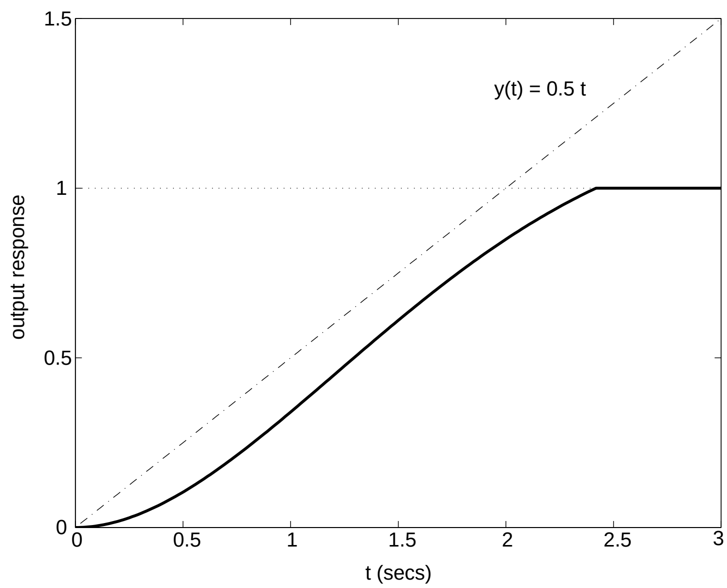

Figure 4: Output response y to a unit-step input for the reset control system.

4 Conclusion

The main contribution of this paper is an example of control specifications that can be achieved by reset control and not by linear feedback. This does not imply that reset control is superior; rather, that reset control has a different set of performance limitations. Such differences can be exploited in specific control applications as demonstrated in [4], [5], [8] and [11].

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