ON QFT TUNING OF MULTIVARIABLE MU CONTROLLERS[1]

J.W. Lee[2], Yossi Chait[3], Maarten Steinbuch[4]

ABSTRACT

Optimal control involves feedback problems with explicit plant data and performance criteria for which a solution is either synthesized or ruled out. H_∞ optimal control is probably the most renowned technique in this class where the control synthesis procedure involves various iterations over weightings. In this paper we argue that the integration of optimal control synthesis and manual tuning in the Quantitative Feedback Theory (QFT) design environment enables design of controllers with levels of performance that surpasses what can be achieved using only a single technique. Specifically, using a constructive example, we demonstrate that QFT's open-loop tuning is can be more transparent than tuning closed-loop weights. In this example, QFT tunes the μ controller with the objective of reducing control bandwidth while maintaining robust performance ($\mu < 1$).

Keywords: control design, robust control, multivariable

INTRODUCTION

In the past decade, norm-based optimal control techniques have taken center stage for solving simple as well as complex control problems in linear, time-invariant feedback systems. They allow for plant descriptions that include various classes of norm-bounded uncertainties in unstructured models. And handle single-loop and multi-loop problems alike. The performance specifications are defined in terms of H_∞ norms of closed-loop transfer functions and optimal control generates the solution if it exists. Control engineers often use experience and insight into a particular problem as the design guidelines and prefer use of manual loop shaping as the means of generating the controller. This approach has the advantage in that the designer can work directly with frequency responses. Moreover, the designer can explicitly invoke practical constraints that are not easily handled in optimal control formulations such as pole location, minimal damping ratio, and controller and stability. And desired modifications about small frequency bands is transparent using QFT's open-loop tuning. Chiefly for such practical issues, the Quantitative Feedback Theory (QFT) has found a following in the industrial control community, especially due to that many control problems are of the SISO type. In QFT, the quality of the design strongly depends on the skills of the control engineer, with respect to manual loop shaping. But it also requires a great deal of experience in complex problems such as multivariable systems with significant interaction. Naturally, the availability of an initial design

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would be of great help to the QFT designer. Based on our extensive experience with both H_∞ and QFT design approaches, it appears that by combing both into a single design process, the control engineer could enjoy the benefits offered by each approach. To illustrate this point, we use a multivariable design problem from the μ toolbox (Balas *et al.*, 1994) involving a robust performance.

Recent work on the relation between QFT(Horowitz, 1992) and H_∞ focused on comparing the resulting designs from the view point of control bandwidth (Chait and Hollot, 1990) and on the conversion of the QFT problem into an H_∞ problem. Zhao and Jayasuriya (1996, 1998) formulated a QFT robust tracking problem with real parameter uncertainty as a mixed-performance H_∞ problem. Using a new robust stability criterion they reduced the inherent conservatism in H_∞ (which treats only complex uncertainty). While our paper also considers these two design techniques, its focus and contribution are different. Specifically, we start with a μ design and then use multivariable QFT to tune the μ controller with the objective of reducing control bandwidth while maintaining robust performance ($\mu < 1$).

Towards a Unified Approach. Considering the weaknesses and strengths of optimal control and QFT, it seems worthwhile to explore the possibilities of integrating the two approaches into a single, sequential, design procedure.). The argument is two-fold. First, as knowledgeable users of μ-synthesis, we have found out that its controllers do not always have all the properties we are looking for. Such controllers may exhibit resonances, instability and have exceedingly high orders. And in some instances, it is difficult to gain insight into performance weights modifications necessary to “tune” the controller. The use of weights with increasing orders for tuning purposes may not be as transparent as QFT tuning which does not involve weights and is executed directly on the open-loop frequency response (Steinbuch *et al*., 1998). With these limitations in mind, an optimal controller, in this paper a μ controller, can be viewed as a natural starting point for a QFT-based tuning. Second, as knowledgeable users of QFT, we realize the difficulties in finding an initial design using manual loopshaping. This is especially acute in complex multivariable problems. Hence, our proposed design philosophy draws on the strengths of both μ-synthesis and QFT techniques.

THE DESIGN PROBLEM

This section describes a design example taken from the μ-analysis and synthesis toolbox (Balas *et al*., 1994). It involves a 2x2-pitch axis controller of an experimental highly maneuverable airplane, HIMAT. In the following, we present only the salient quantitative features of the design problem. The interested reader should refer to Balas *et al*. (1994) for additional insight into the problem and the design via the μ technique. The block diagram of the HIMAT control problem is shown in Fig. 1.

The nominal plant transfer matrix G_{nom} corresponding to the airplane dynamics is given in its state-space data

$$G_{nom}: \quad \begin{bmatrix} A_{4\times4} & B_{2\times4} \\ C_{4\times2} & D_{2\times2} \end{bmatrix} = \begin{bmatrix} -0.023 & -37 & -19 & -32 & 0 & 0 \\ 0 & -19 & 0.98 & 0 & -0.41 & 0 \\ 0.012 & -0.12 & -2.6 & 0 & -78 & 22 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 57 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 57 & 0 & 0 \end{bmatrix}$$

This plant exhibits significant non-diagonal dynamics. The actual plant dynamics is not known precisely, however, for design purposes it is represented by the family $\mathsf{G} = \{G_{nom}(I + \Delta_G W_{del}): \quad \Delta_G \text{ stable}, \|\Delta_G\|_\infty < 1\}$ where the uncertainty weight W_{del} is given by $W_{del} = w_{del} I_{2\times2}, w_{del}(s) = \frac{50(s+100)}{s+10000}$. The robust stability problem is to find the controller K that achieves nominal closed-loop internal stability and satisfies $\left\| W_{del} K G_{nom} (I + K G_{nom})^{-1} \right\|_\infty < 1$. The robust performance problem is to design a controller K that achieves closed-loop internal stability for all $G \in \mathsf{G}$, and that the output sensitivity performance objective $\left\| W_P (I + GK)^{-1} \right\|_\infty < 1$ is satisfied for each $G \in \mathsf{G}$, where the performance weight is given by $W_P = w_P I_{2\times2}, \ w_P(s) = \frac{0.5s+1.8}{s+0.018}$

μ SYNTHESIS

The initial step involved in μ synthesis using the toolbox (Balas *et al.*, 1994) is the construction of the open-loop interconnection structure followed by the closed-loop linear fractional transformation. The design step involves so-called D-K iteration. After 3 D-K iterations using dynamic D-scales, a 28th order control was synthesized which achieves robust performance. Using truncated balanced realization (Balas *et al*., 1994), it was subsequently reduced to 12th order. As is the case in practically every industrial implementation, it is desired to reduce the high-frequency control gains as rapidly possible subject to robust performance constraint. For this purpose, sensor noise signal was included (see open-loop interconnection diagram in Fig. 2).

In the next section we show how MIMO QFT can be used to reduce the high frequency control gains without the addition of sensor noise minimization constraint. The sensor noise weight is given by

$$W_n = w_n I_{2\times2}, \quad w_n(s) = \frac{2s+2.56}{s+320}$$

Again, using 3 D-K iterations with dynamic D-scales, a 26th-order controller was synthesized which achieves robust performance. Using truncated balanced realization, it was subsequently reduced to 12th order. The plots of the structured singular value test, μ tests, for the two designs (with and without sensor noise weight) are shown in Fig. 3. As expected, adding the noise weight resulted in a lower bandwidth controller (Fig. 4).

At this stage, if we desire further bandwidth reduction, it would appear that the order of the weights, especially W_n, must be increased to exploit available "flexibility" in the crossover range. While it is possible to perform tuning this way, the next section present a new tuning approach that is more transparent for the engineer. With this in mind, we turn our attention to MIMO QFT. Specifically, we will demonstrate how MIMO QFT is used to solve the original robust performance problem while explicitly attempting to minimize high-frequency control gains.

QFT TUNING

The purpose of QFT tuning in this context is to study possible design tradeoffs in this problem without making explicit modifications to the weights as required in μ synthesis. The issue of minimization of the high frequency gain is dealt with in QFT in an indirect way. The basic idea is that hard performance specifications (such as sensitivity reduction) are considered only up to some crossover frequency. The individual elements of a diagonal controller are designed sequentially to satisfy the MIMO performance constraints. Unlike the approach used in optimal control, the high-frequency responses of the diagonal controllers are typically not constrained a priori; and are minimized during QFT loopshaping. We will then compare the QFT's high frequency gains to those of the μ design.

This QFT design is done as follows. A diagonal QFT controller, $F = \mathit{diag}[f_1, f_2]$, is inserted just before the μ controller K. The robust stability constraint becomes $\left\| W_{del} KFG_{nom} (I + KFG_{nom})^{-1} \right\|_\infty < 1$, and the robust performance constraint is that $\left\| W_P (I + GKF)^{-1} \right\|_\infty < 1$ is satisfied for each $G \in \mathcal{G}$.

The technical issue we must deal with now is that the QFT design framework for MIMO systems is inherently different from norm-based approaches. In QFT, performance specifications are placed on each SISO element in the matrix function of interest. It is impossible for QFT to deal direclty with norm-based specifications. However, as we show below, we believe that for the purpose of controller tuning, it is possible to modify the weights from a norm-based formulation into the QFT's framework and still maintain the basic performance requirements.

We first modify the full block uncertainty Δ_G considered in (1) as follows. Consider a block diagonal structure $\mathit{diag}\left[\Lambda_1(s), \Lambda_2(s)\right]$ and approximate the frequency responses of the families Λ_i using an N-point representation of their boundaries $\tilde{\Lambda}_i(j\omega) = \left\{\cos(n\pi/N) + j\sin(n\pi/N): \quad n = 1, \ldots, N\right\} \subseteq \Lambda_i(j\omega), \; i = 1,2$, resulting in an approximated frequency response set of the uncertainty $\tilde{\Delta}_G(j\omega) = \mathit{diag}\left[\tilde{\Lambda}_1(j\omega), \tilde{\Lambda}_2(j\omega)\right]$. The approximate plant family becomes $\tilde{\mathcal{G}} = \left\{G = G_{nom}(I + \Delta_G W_{del}): \; \Delta_G(j\omega) \in \tilde{\Delta}_G(j\omega), \Delta_G \text{ stable}\right\}$.

The nominal plant remains unchanged and this approximate plant family consists of (N+1)x(N+1) members. Let us now define a new plant P consisting of the original plant G cascaded with the μ-controller K, P = GK. The new plant family is $\tilde{\mathcal{P}} = \{P = GK : G \in \tilde{\mathcal{G}}\}$. With the QFT controller F, the robust output sensitivity specification becomes $\|W_P S\|_\infty < 1$, for each $P \in \tilde{\mathcal{P}}$ where $S = (I+PF)^{-1}$. In MIMO QFT, performance specifications are placed on each element of the sensitivity transfer function matrix. The norm-based above specification cannot be translated into this format without conservatism. However, our focus is to tune the μ controller K so to minimize its high frequency gains while maintaining its low-frequency performance. With this in mind, we suggest the following workaround. For each $P \in \tilde{\mathcal{P}}$ we compute the sensitivity with nominal QFT control (i.e., F = I),

$$S_m = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \frac{\begin{bmatrix} 1+p_{22} & -p_{12} \\ -p_{21} & 1+p_{11} \end{bmatrix}}{1+p_{11}+p_{22}+\det[P]}, \quad P_i = \{p_{ij}\}_m \in \tilde{\mathcal{P}}, \ m = 1,\ldots,(N+1)^2$$

and use it as the baseline frequency response for QFT tuning. That is, since the μ controller K already satisfies our performance specification, the magnitude of the μ sensitivity for each plant in the approximate family $W_m = |S_m|$, $m = 1,\ldots,(N+1)^2$ (we use the notation $W_m = \lfloor w_{ij_m} \rfloor$) is utilized to define the following QFT's robust performance problem. With the QFT controller $F = \mathrm{diag}[f_1, f_2]$ included, the sensitivity transfer function becomes

$$S_m = \begin{bmatrix} s_{11_m} & s_{12_m} \\ s_{21_m} & s_{22_m} \end{bmatrix} = \frac{\begin{bmatrix} 1+p_{22_m} f_2 & -p_{12_m} f_2 \\ -p_{21_m} f_1 & 1+p_{11_m} f_1 \end{bmatrix}}{1+p_{11_m} f_1 + p_{22_m} f_2 + \det[P_m] f_1 f_2}, \ m = 1,\ldots,(N+1)^2$$

and for each plant in the family $\tilde{\mathcal{P}}$ we write the sensitivity specification as

$$\left|s_{ij_m}\right| \le w_{ij_m}, \ i, j = 1,2, \ m = 1,\ldots,(N+1)^2$$

The idea here is that if we satisfy the above inequalities, or stay near them, the diagonal scaling created by the QFT controller F will not affect the original norm-based constraint $\left\|W_P (I + GKF)^{-1}\right\|_\infty < 1$ which is met with F = I. This will hold true as long as tuning is not expected to radically modify the open-loop responses. While robust stability of the design is automatically guaranteed in optimal control, in QFT one must include this constraint explicitly. Specifically, the robust stability problem is stated as follows. Assuming no unstable pole/zero cancellations in the loop, the feedback system is robust stable if the nominal system is stable and $|I + P(j\omega)F(j\omega)| > 0$, for each $P \in \tilde{\mathcal{P}}$ or for the approximate plant $|I + P_m(j\omega)F(j\omega)| > 0$, $m = 1,..,(N+1)^2$. Clearly, robust stability with our approximate plant does not imply robust stability with the original plant even if $N \to \infty$. However, with sufficient MIMO stability margins enforced in the QFT tuning, the risk is minimized. In addition, we will analyze the resulting design using μ analysis. To this end, the QFT robust stability margin specification takes on the form (Yaniv, 1995)

$$\left|s_{ii_m}\right| \le \max\left|\frac{1}{w_P}\right|,\ i=1,2,\ m=1,\dots,(N+1)^2$$

where

$$s_{ii_m} = \frac{1}{1+p^e_{ii_m} f_i}$$

and where

$$p^e_{11_m} = \frac{p_{11m} - \det[P_m] f_2}{1+p_{22_m} f_2}, \quad p^e_{22_m} = \frac{p_{22m} - \det[P_m] f_1}{1+p_{11_m} f_1}$$

In the initial tuning step, F =I, and we elect to start with tuning of the first loop. That is, we fix f_2 = 1, and compute QFT bounds for the following set of six algebraic problems which are all bilinear in f_1.

$$\begin{cases} \left|s_{11_m}\right| \le w_{11_m} \\ \left|s_{12_m}\right| \le w_{12_m} \\ \left|s_{21_m}\right| \le w_{21_m} \\ \left|s_{21_m}\right| \le w_{22_m} \\ \left|\dfrac{1}{1+p^e_{11_m} f_1}\right| \le 2 \\ \left|\dfrac{1}{1+p^e_{22_m} f_2}\right| \le 2 \end{cases}, \quad m=1,\dots,(N+1)^2 \tag{4}$$

where have used $\max\left|1/w_P\right| = 2$. Clearly, all the functions in (4) are bilinear in f_1. The QFT Toolbox (Borghesani *et al*., 1994) is used to generate the corresponding bounds at a set of frequencies. These bounds are then intersected to yield working bounds. The controller f_1 is then designed interactively using the QFT Toolbox (Borghesani *et al*., 1994). To achieve nominal stability we actually loopshape $p^e_{110} f_1$ with the nominal plant p^e_{110} corresponding to G_0. A screen capture of a typical interactive loopshaping environment is shown in Fig. 5. Specifically, two QFT bounds, the original (f_1 = 1) and the tuned nominal loops are shown (in tuning we used bounds at 20 frequencies which are not shown here for clarity of the figure). The effect of a 3rd order f_1 on the loop response is highlighted at ω = 500 (see arrow). Naturally, tuning using a higher-order controller can potentially lead to further bandwidth saving, but this tradeoff is really up the designer to make in a specific design.

After f_1 is designed, we proceed to tune f_2. Again, the set of six inequalities (4) is solved to compute QFT bounds in terms of f_2. Again, to achieve nominal stability we actually loopshape $p^e_{220} f_2$ with the nominal plant p^e_{220} corresponding to G_0. A screen capture of a typical interactive loopshaping environment is shown in Fig. 6. Specifically, two QFT bounds, the original ($f_1 = 1$) and the tuned nominal loops are shown. The effect of a 5th order f_2 on the loop response is highlighted at $\omega = 300$ (see arrow).

Because we have been tuning the approximate plant only, at each step we analyze the structured singular values using the μ toolbox. As it turns out, using a few QFT iterations, we were able to find the direction μ changes for small changes in the open-loop response. This is especially useful when we tune the response over a "small" frequency band. That insight is exactly what makes QFT tuning so powerful. It is important to note that the QFT performance bounds are not exact relative to the original, norm-based specifications. And so, it is feasible that the nominal loop does not satisfy its bounds, yet the structured singular value is below 1. This insight is learned during the QFT tuning/μ analysis tuning cycle.

At each step, the SISO controller was order-reduced (Wortelboer and Bosgra, 1992) using the QFT Toolbox (Borghesani *et al*., 1994). In Figs 5-6, one can observe what QFT can offer in terms of reducing the high-frequency gain. While satisfying the low-frequency robust performance bounds (the line across the Nichols chart), and avoiding the robust stability margins bound (the closed curve in Nichols chart), one can attempt to reduce high frequency gain by adding/tuning any number of "far-off" poles. The designer can tune the values of such poles by interactively dragging the loop response to the left/down at a specific frequency. The feasible limit for such shifts is exactly the QFT bound. This is a rather straightforward process yet it does require experience. The Bode plots of the QFT controller are shown in Fig. 7. We can observe that $|f_i| < 1$ (except for narrow band near 20 Hz) with bandwidths below 10 kHz. Particularly impressive is the fact the using QFT tuning we were also able to reduce the low-frequency control gain (i.e., approaching $\mu = 1$).

The final design consists of the cascaded GF controller. Using truncated balanced realization the order of GF is reduced to 12. Figure 8 shows the structured singular values of the 12th-order μ design (with sensor noise) and the 12th-order μ/QFT design (without noise). For fair comparison, the structured singular values for the μ/QFT design were computed using the interconnection with sensor noise.

Finally, we compare the resulting reduction in the controller high frequency gains between the two design approaches. Figure 9 depicts such values for the 12th-order combined QFT/μ design (without sensor noise), and the 12th-order μ design with sensor noise. It is interesting to note that while QFT tuning, we observed that the structured singular values in Fig. 3 were below one, and so we used a DC gain of 0.85 in the QFT's SISO controllers f_i. As seen, the integrated μ/QFT design satisfies the robust performance with the lowest bandwidth.

A final comment for QFT experts who may wonder about the required complexity of the controller, it is instructive to inspect the coupling in the μ design. The Bode plots of the individual elements in the μ/QFT controller indicate a high-degree of interaction (Fig. 10). While it is conceivable that a static decoupler ($\omega = 0$)can be easily designed, the dynamic nature of G in the range $\omega \in [1,100]$ clearly indicates that it will be very difficult to manually design it with the same ease offered by optimal control.

CONCLUSIONS

In this paper we have shown using a generic, multivariable, robust performance problem, that the integration of μ-synthesis and QFT tuning led to a controller whose performance levels may not be achievable if only a single technique was used. This design approach enjoys the strength of μ-synthesis in dealing with complex multivariable problems (such as non-square and/or highly coupled plant) and QFT's ability to deal directly with plant frequency response plant and easily tune control response over narrow frequency bands. Our findings strongly suggest that the historical academic competition between classical and modern optimal design philosophies should end. And it motivates development of new graphical tools that seamlessly implement the integration of μ-synthesis and QFT tuning.

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LIST OF FIGURES

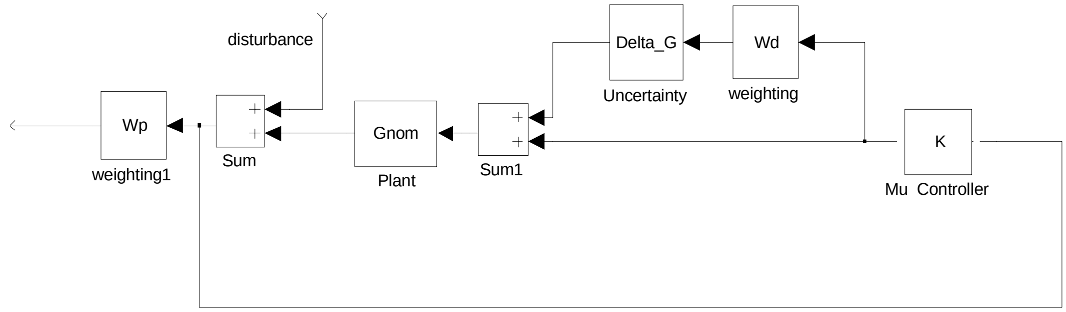

Figure 1: Closed-loop interconnection structure of HIMAT

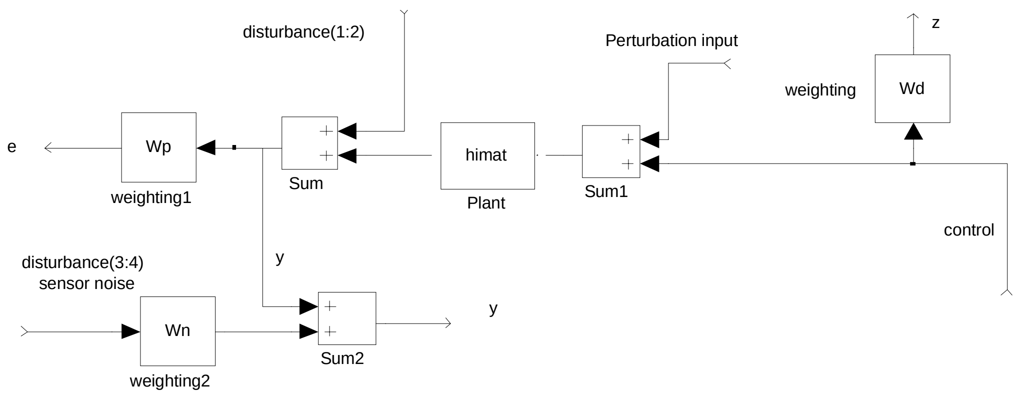

Figure 2: Open-loop interconnection structure of HIMAT with sensor noise

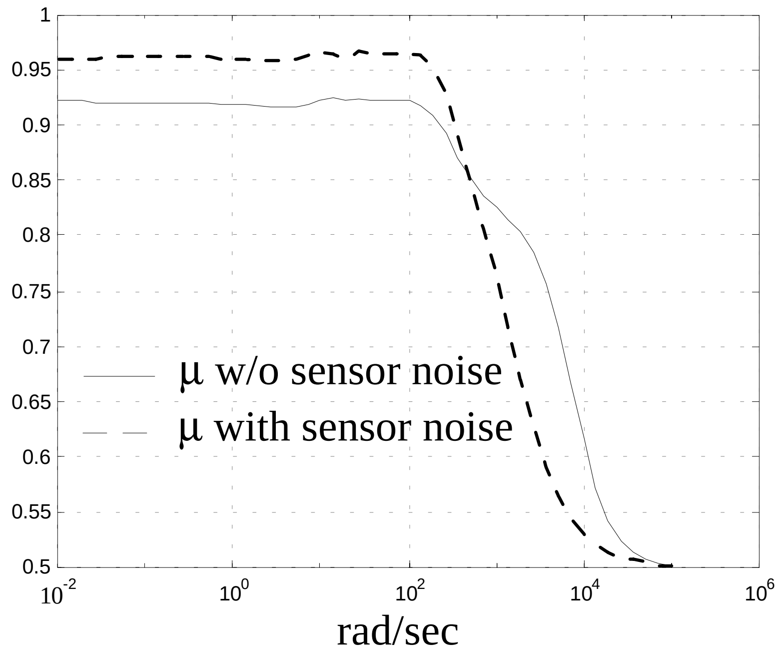

Figure 3: Structured singular values of the two μ designs

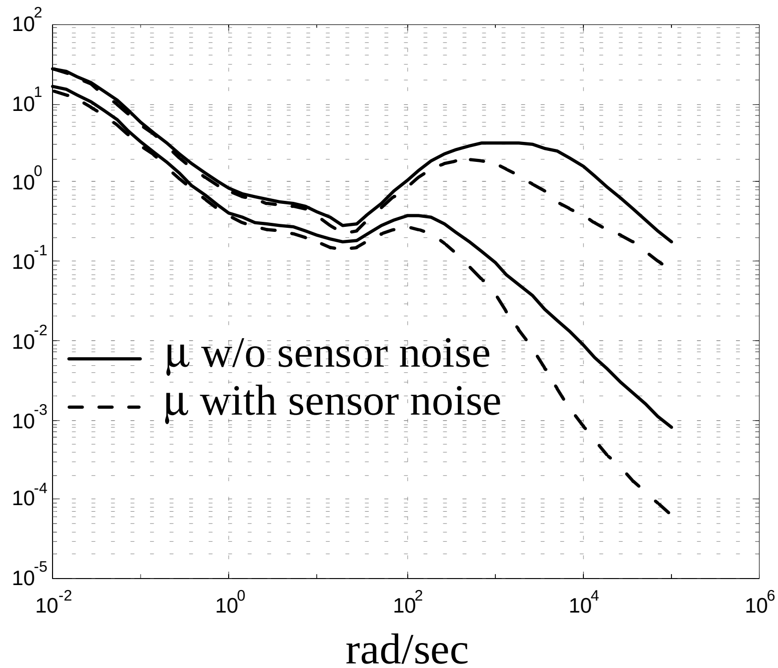

Figure 4: Singular values of the two μ controllers

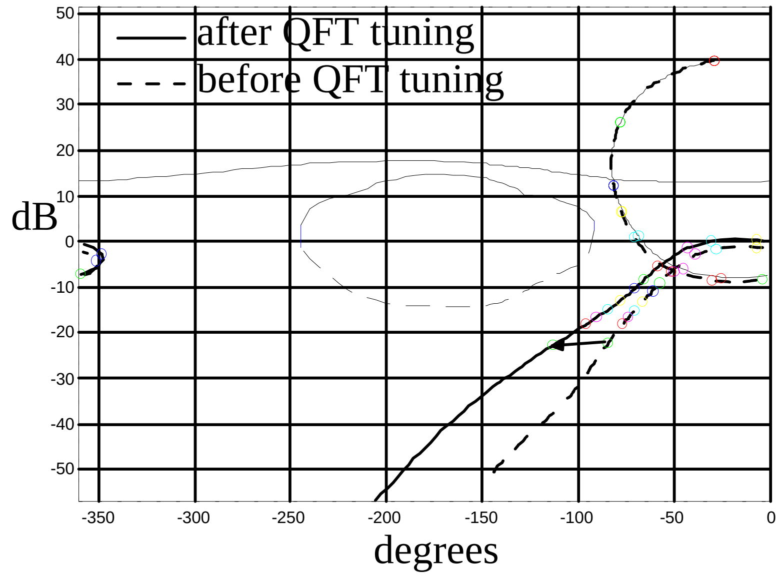

Figure 5: QFT bounds, tuned and un-tuned nominal loops in the first loop

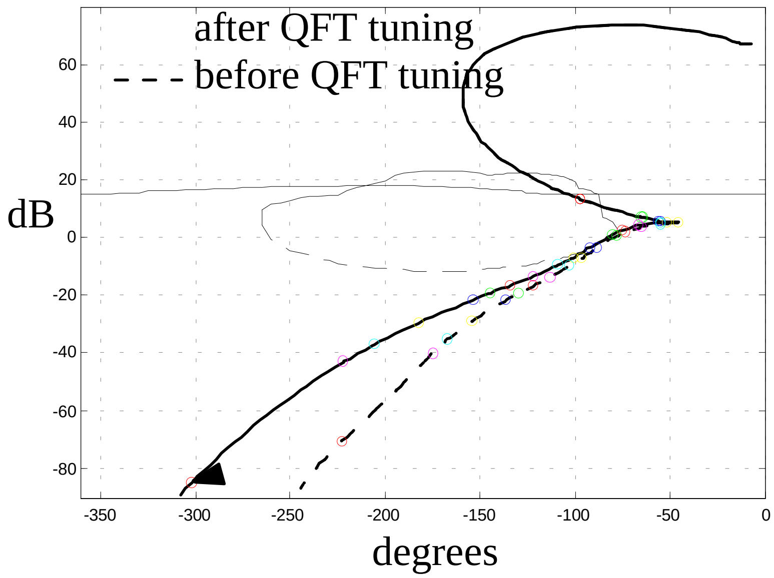

Figure 6: QFT bounds and final design for the second loop

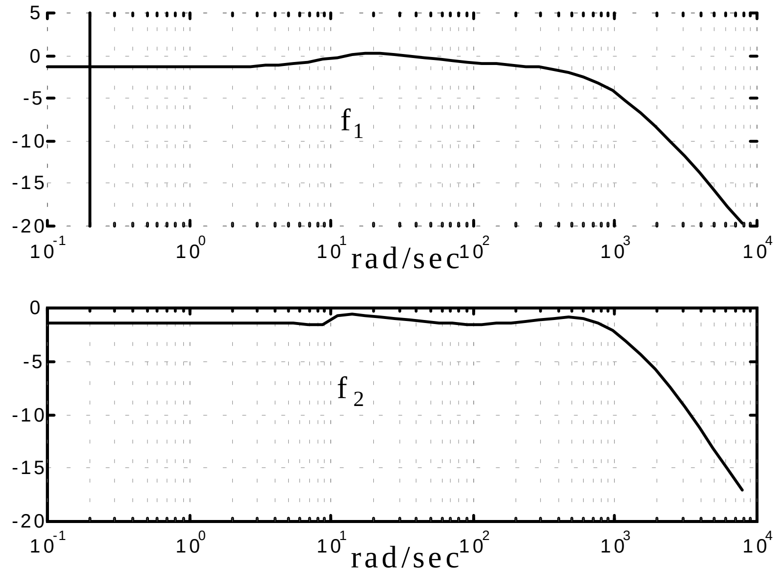

Figure 7: Bode plots of the diagonal QFT controller

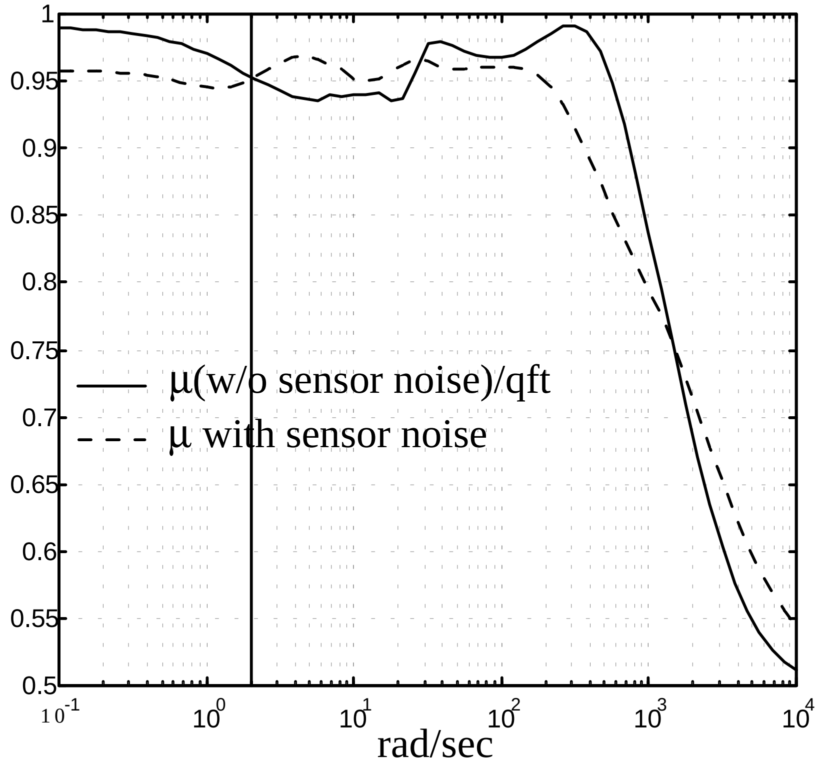

Figure 8: Comparison of structured singular values of two design approaches

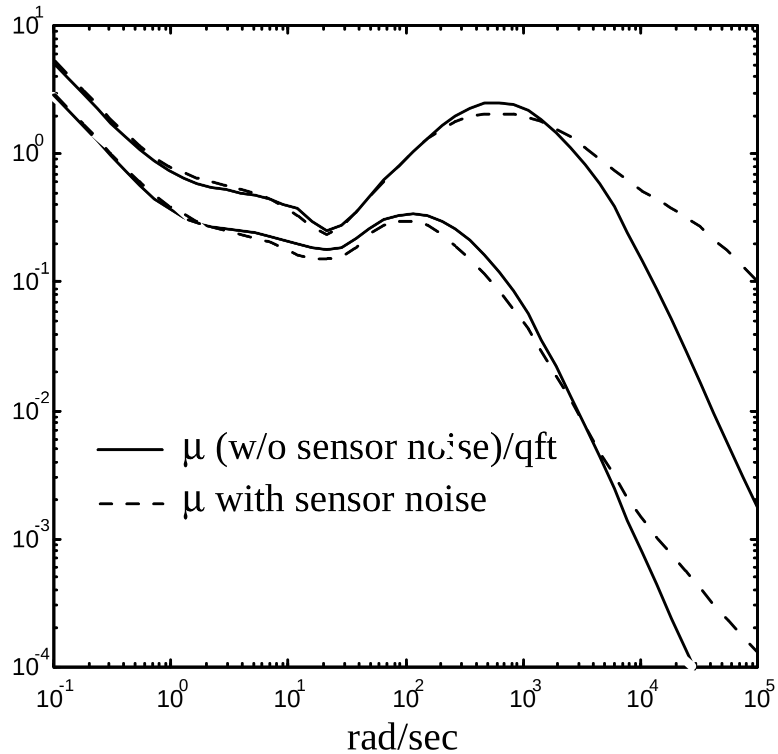

Figure 9: Comparing controller singular values of the two design approaches

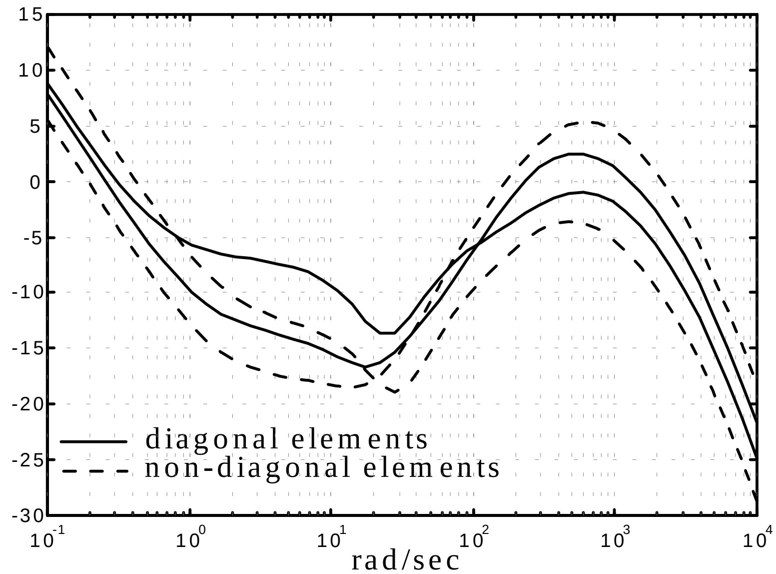

Figure 10: Magnitude frequency responses of the SISO elements in the μ/QFT controller