BIBO Stability of A Class of Reset Control System

Qian Chen‡, C.V. Hollot† and Yossi Chait‡

Abstract

A reset element is a linear system whose states are reset to zero whenever its input meets a threshold. Such elements were introduced in feedback control systems with the aim of providing more favorable performance trade-offs than could be achieved by LTI control. This potential has been demonstrated in some recent experiments, but there is no guarantee of BIBO stability. This paper develops a sufficient condition for BIBO stability and applies it to an experimental reset control system of nontrivial complexity.

1 Problem Description

The structure of the reset control systems under consideration is shown in Figure 1, where $L(s)$ denotes the linear plant and signals $r(t)$, $y(t)$, $e(t)$, $n(t)$ and $d(t)$ represent reference input, output, error, sensor noise and disturbance, respectively. The first-order reset element (FORE), introduced in [1], is described by the first-order impulsive differential equation:

$$\begin{aligned} \dot{x}_c(t) &= -bx_c(t) + e(t); & e(t) \neq 0 \\ x_c(t^+) &= 0; & e(t) = 0 \end{aligned} \tag{1}$$

where b is FORE's pole and $x_c(t)$ is its state. The time instants when $e(t) = 0$ are called reset times. We assume that these reset times t_i can be collected into the set

$$I = \{t_i \mid e(t_i) = 0,\ t_i > t_{i-1} + \sigma,\ \sigma > 0,\ i = 1, 2, ...\}.$$

This definition implies that the interval between any two adjacent reset actions is greater than σ. Assume that $\{A, B, C\}$ is a minimal realization of $L(s)$ and $x_p(t) \in \Re^n$ is the plant states. The state-space description of the reset control system in Figure 1 is

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bx_c(t) & \\ \dot{x}_c(t) &= -Cx_p(t) - bx_c(t) + w(t); & t \notin I \\ x_c(t^+) &= 0; & t \in I \end{aligned} \tag{2}$$

where $w(t) = r(t) - n(t) - d(t)$ and $y(t) = Cx_p(t) + d(t)$.

[0]Supported by the NSF under Grant No. 9800612.

‡ MIE Dept, UMASS-Amherst, MA 01003.
† ECE Dept, UMASS-Amherst, MA 01003.
Email: {qchen, hollot, chait}@ecs.umass.edu

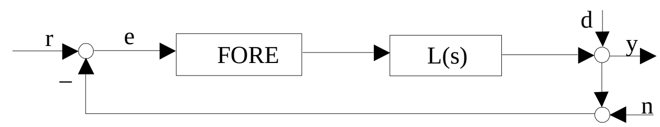

Figure 1: Block diagram of reset control system

2 BIBO Stability

Our main result is as follows:

Theorem 1 Let $A_{cl} = \begin{bmatrix} A & B \\ -C & -b \end{bmatrix}$. If there exists a $\beta \in \Re$ such that the transfer function

$$h(s) \triangleq [\beta C \quad 1](sI - A_{cl})^{-1}[0 \ \cdots \ 0 \ 1]^T$$

is strictly positive-real (SPR) and has no zero-pole cancellation, then system(2) is BIBO stable.

Example: We now apply the stability criteria in Theorem 1 to the experimental reset control system reported in [2]. This reset control design achieved good performance in a tape-speed control system, but there was no formal proof of stability. The structure of this control system is that shown in Figure 1 where the FORE's pole is $b = 30$ and the linear plant transfer function $L(s)$ is:

$$\frac{5924839(s+30)(s+257)(s+3.6)(s^2+20s+216)}{s(s+220)(s+14)(s+37)(s^2+14s+62)(s^2+88s+5925)} \frac{(s^2+56s+3679)(s^2+76s+15790)(s^2-696s+216100)}{(s^2+75s+20030)(s^2+203s+32800)(s^2+126s+15790)}.$$

It is straightforward to show that the associated $h(s)$ is SPR for $\beta = 0.1$. Hence, the experimental reset control system in [2] is verified to be BIBO stable.

References

[1] I. Horowitz and P.Rosenbaum, "Nonlinear design for Cost of Feedback Reduction in systems with Large Parameter Uncertainty," International Journal of Control, Vol. 24, No. 6, pp. 977-1001, 1975.

[2] Y. Zheng, Theory and Practical Considerations in Reset Control Design, Ph.D. Dissertation, University of Massachusetts, Amherst, 1998.