A "NEW" APPROACH TO ACTIVE NOISE CONTROL IN DUCTS[1]

Prashant Mehta[2], Yuan Zheng[3], Yossi Chait[4] [5] and C.V. Hollot[6]
University of Massachusetts
Amherst, MA 01003

Abstract

In this paper we consider the fixed-filter design approach for active noise control (ANC) in ducts. Based on a large body of literature during the past decade that deals strictly with adaptive filter solutions (see [1] and references contained therein) the fixed-filter approach appears to be all but discarded for this application. The ANC system under consideration has a similar configuration to those used in most adaptive implementations; i.e., a duct equipped with both feedforward and feedback sensors and a control speaker. Our design of a linear, time-invariant controller involves three steps: (1) system identification from the experimental duct frequency response, (2) H_∞ synthesis using the identified plant and model errors, and (3) use of Quantitative Feedback Theory (QFT) to further improve performance by sequential tuning of the feedback and feedforward filters using the experimental data. Preliminary experiments indicate that attenuation levels of 10–30 dB are achievable over a 400 Hz band without amplification in the immediate range. An important consequence of LTI designs is that performance is guaranteed for wide-band noise as well as pure tone disturbances – in adaptive schemes stability and performance may be very difficult (if impossible) to guarantee beforehand. This paper focuses on the third step of our three-step approach – the QFT design aspect – (the other two steps are described in [2]).

Introduction

Historically, ANC in ducts has been the focus of much attention since successful duct silencing has a commercial potential [3]. For example, active noise reduction in automobile exhaust systems reduces back pressure on the engine thereby increasing engine efficiency as well as attenuating low-frequency engine noise that traditional mufflers fail to quiet. There exists a potential for reducing acoustic pollution by attenuating fan and blower noise in HVAC systems. Indeed, the 50 – 500 Hz range includes the throb and rumble of turbulent airflow and the roar, hum, buzz and whine of fans, pumps and chillers. Hence, the motivation for successful ANC systems is well recognized in both industry and academia.

The state-of-the-art in ANC in ducts uses adaptive feedforward algorithms primarily derived from adaptive noise cancellation techniques; see [4] for an introduction to adaptive filtering with application to ANC. Despite the positive outlook in a recent ANC review paper [1], adaptive schemes suffer drawbacks including instability [5] especially in the presence of acoustic feedback which is prevalent in duct systems; see [3]. A natural approach to these adaptive schemes is fixed-filter designs which enjoy tractable performance and stability analysis. However, the single-loop feedback schemes typifying much of the past research in this area display modest levels of noise cancellation when compared to the adaptive

[1] Research supported in part by an NSF grant No. MSS-9313764
[2] Graduate Research Assistant, ECE Department, Now is with Control Group, United Technology Research Center MS-29, UTRC, 411 Silver Lane, East Hartford, CT 06108, mehtapg@utrc.utc.com
[3] Graduate Research Assistant, ME Department, zheng@ecs.umass.edu
[4] Associate Professor, ME Department, chait@ecs.umass.edu
[5] Author to whom all correspondence should be made to
[6] Associate Professor, ECE Department, hollot@ecs.umass.edu

approach. This paper focuses on a fixed-filter strategy employing both feedforward and feedback filters which is closer in control architecture to the adaptive techniques and which achieves (in experiments) comparable levels of noise cancellation.

Our approach for ANC control design of a linear, time-invariant controller involves three steps: (1) system identification from the experimental duct frequency response, (2) H_∞ synthesis using the identified plant and model errors, and (3) use of Quantitative Feedback Theory (QFT) to further improve performance by sequential tuning of the feedback and feedforward filters using the experimental data. The ANC system under consideration has a similar configuration to those used in most adaptive implementations; i.e., a duct equipped with both feedforward and feedback sensors and a control speaker. An overview of the three design steps as applied to the above ANC system can found in [2]. This paper focuses more deeply on the QFT design step [2].

This paper is organized as follows. Section 2 defines the ANC control design problem. Section 3 discusses briefly how an initial design was obtained using the H_∞ method. Section 4 describes the QFT controller tuning steps. Section 5 includes description of the experimental setup and results.

The Design Problem

The experimental setup for the feedforward/feedback ANC configuration (Figure 1) consists of a disturbance speaker (signal d), a control speaker (signal u), a detection microphone (output m) located upstream from the control speaker and an error microphone (output e) located downstream. The transfer functions from the disturbance and the control speakers to the error and detection microphones are denoted by $P_{ed}(s)$, $P_{eu}(s)$, $P_{md}(s)$ and $P_{mu}(s)$, respectively. All relevant hardware, such as speakers, microphones, power amplifier and microphone preamps are included in these transfer functions. The magnitude of the measured frequency responses are shown in Figure 2.

The objective is to design a control law $u(s) = C_m(s)m(s) + C_e(s)e(s)$, where $C_m(s)$ and $C_e(s)$ are respectively the feedforward and feedback filters such that the magnitude of the closed loop transfer function from $d(s)$ to $e(s)$:

$$\frac{e(s)}{d(s)} = \frac{C_m(s)(P_{mu}(s)P_{ed}(s) - P_{eu}(s)P_{md}(s)) + P_{ed}(s)}{1 + C_m(s)P_{mu}(s) + C_e(s)P_{eu}(s)} \tag{1}$$

is small in the attenuation band with limited amplification elsewhere. In this study, we focus our attenuation in the 100 – 500 Hz frequency band which is of primary interest in duct and many other noise control problems.

The H_∞ Synthesis Step

The QFT design procedure can be applied to single-loop as well as to multi-loop controllers. In multi-loop cases, the design focuses on each single-loop controller at a time. In certain multi-loop problems that require complex controllers due to the presence of a large number of highly underdamped resonances, the QFT design process is rather difficult to initiate. In ANC in ducts this difficulty is exacerbated since it is impossible to decouple the design of the feedforward and the feedback controllers which both affect closed-loop stability. Whether the feedforward or the feedback controller is designed first, it is difficult to decide how to distribute the load (i.e., bandwidth and complexity) between the two controllers, when each controller may consist of 10 poles packed into a decade of frequencies. The presence of right-half plane zeros in ANC dynamics makes the problem even more challenging. In order to overcome these difficulties, we use a modern control method (H_∞) to obtain a first-cut design which is then fine-tuned with QFT. The H_∞ design step does not work directly on the experimental frequency response data and requires a finite-dimensional LTI model of the process. As a result, the modelling errors between the real data and an identified LTI system (such errors are non-trivial in vibratory systems such as acoustic ducts) must be taken into account. One of the drawbacks of the H_∞ approach is that such errors are modelled by unstructured, norm-bounded uncertainty which may be overly conservative, leading to poorly-performing ANC designs. This is where QFT plays a complementary tuning role since it is non-model based, dealing directly with the experimental data.

The disturbance attenuation problem for

both H_∞ and QFT designs is

$$\left|\frac{e(j\omega)}{d(j\omega)}\right| \le \left|\frac{1}{50\frac{.88\frac{j\omega}{2\pi 100}}{(\frac{j\omega}{2\pi 100}+1)(\frac{j\omega}{2\pi 600}+1)}}\right| \qquad (2)$$

The modelling errors in $P_{eu}(s)$ and $P_{mu}(s)$ are represented by additive, norm-bounded uncertainties; see [2] and [6] for details. Each of the four open-loop functions were augmented with a first-order Padé approximation $\frac{1-0.0005s}{1+0.0005s}$ to account for the 0.1 *msec* sample-time delay associated with the real-time implementation of the control system.

For the H_∞-synthesis step, a finite-dimensional state-space model of the duct was identified from the experimental frequency responses using the Chebyshev polynomial curve-fitting approach described in [7] and [8]. Full details of the identification step and the H_∞ synthesis are available in [6] and [2]. Control design was accomplished using the Robust Control Toolbox [9].

QFT tuning

In this section we see an interplay between the H_∞ and QFT design techniques. While H_∞ is model-based and hence must use (possibly conservative) error models, QFT tuning works directly with the experimental frequency response data. Also, in the H_∞ synthesis, the feedforward/feedback controllers are designed in a single shot, whereas QFT designs them one at a time. In the first step of QFT tuning, $C_e(s)$ is fixed and the feedforward controller $C_m(s)$ is tuned, and in the second step, $C_m(s)$ is fixed and $C_e(s)$ is tuned.

Design Equations

In the first design step, it is possible to re-write the closed loop function from the disturbance $d(j\omega)$ to the error microphone output $e(j\omega)$ as

$$\frac{C_m(j\omega)\frac{B(j\omega)}{1+C_e(j\omega)P_{eu}(j\omega)}+\frac{P_{ed}(j\omega)}{1+C_e(j\omega)P_{eu}(j\omega)}}{1+C_m(j\omega)\frac{P_{mu}(j\omega)}{1+C_e(j\omega)P_{eu}(j\omega)}} \qquad (3)$$

where,

$$B(j\omega) = P_{mu}(j\omega)P_{ed}(j\omega) - P_{eu}(j\omega)P_{md}(j\omega) \qquad (4)$$

with the control objective given in (2). The performance problem in Eqs. (2) and (3) is in standard linear fractional transformation form for computing QFT bounds [10]. The associated loop is

$$L_m(j\omega) = C_m(j\omega)\frac{P_{mu}(j\omega)}{1+C_e(j\omega)P_{eu}(j\omega)} \qquad (5)$$

Once $C_m(s)$ is re-designed, QFT-based tuning is accomplished on the feedback controller $C_e(s)$. In this case, the function from $d(j\omega)$ to $e(j\omega)$ is

$$\frac{\frac{P_{ed}(j\omega)}{1+C_m(j\omega)P_{mu}(j\omega)}+C_m(j\omega)\frac{B(j\omega)}{1+C_m(j\omega)P_{mu}(j\omega)}}{1+C_e(j\omega)\frac{P_{eu}(j\omega)}{1+C_m(j\omega)P_{mu}(j\omega)}} \qquad (6)$$

where $B(j\omega)$ is given in (4). The corresponding loop is

$$L_e(j\omega) = C_e(j\omega)\frac{P_{eu}(j\omega)}{1+C_m(j\omega)P_{mu}(j\omega)} \qquad (7)$$

where $C_m(s)$ is the one designed in the first QFT design step. Again, the objective remains the same; i.e., stabilize and meet (2).

Stability Issues

Consider the ANC system in Figure 1. In the QFT steps, only stable controllers $C_e(s)$ and $C_m(s)$ are designed. Also, since the duct is a passive system, the transfer functions $P_{eu}(s)$ and $P_{mu}(s)$ are stable. Thus, this control system is stable if the Nyquist plot of

$$1 + C_m(s)P_{mu}(s) + C_e(s)P_{eu}(s) \qquad (8)$$

does not encircle the origin. Now, in the first QFT step, $C_e(s)$ is frozen and $C_m(s)$ is designed. We express (8) as a stabilizability constraint on $C_m(s)$, and to this end rewrite (8) as

$$(1+C_e(s)P_{eu}(s))\left(1+C_m(s)\frac{P_{mu}(s)}{1+C_e(s)P_{eu}(s)}\right) \qquad (9)$$

Assume that $1 + C_e(s)P_{eu}(s)$ has n RHP zeroes. Then, using (9), a straightforward argument shows that stability is ensured if the Nyquist plot of

$$C_m(s)\frac{P_{mu}(s)}{1+C_e(s)P_{eu}(s)} \qquad (10)$$

encircles *ccw* the critical point $(-1, 0)$ n times. This is the desired stability constraint on $C_m(s)$. Note that if $1 + C_e(s)P_{eu}(s)$ is unstable, then one must take care to induce the correct encirclements. Similarly, in the design of $C_e(s)$, one must ensure that the Nyquist plot of

$$1 + C_e(s)\frac{P_{eu}(s)}{1+C_m(s)P_{mu}(s)} \qquad (11)$$

satisfies the encirclement criterion.

Loop-Shaping Details

To achieve reasonable performance in the initial H_∞ design, the model-error bounds were taken smaller than their actual values. As a result, the H_∞ design was unstable when applied to the experimental frequency response data. The associated Nichols plot of $C_m(s)P_{mu}(s) + C_e(s)P_{eu}(s)$ is given in Figure 3 where a destabilizing encirclement at high-frequency is clearly seen. Thus, in addition to improving performance, the QFT tuning step must stabilize the closed loop. This was accomplished by reducing the bandwidth of both $C_m(s)$ and $C_e(s)$ to remove this destabilizing encirclement (see Remark 4 below). Several cycles of tuning were required to stabilize and improve performance. The loop-shaping environments for $L_m(s)$ and $L_e(s)$ and a few bounds are shown in Figures 4-5. The notation used for plotting QFT bounds is: a solid line indicates that the loop response must lie above the line, while a dashed line indicates that the loop response must lie below the line. The QFT design was executed using the QFT Toolbox [10].

Remarks

1. During the design process it was observed that good performance was correlated to having either $C_m(s)P_{mu}(s)$ or $C_m(s)P_{mu}(s)$ having *ccw* encirclements of the critical point. The reason for this is not yet clearly understood. Note that such encirclements do not imply that the system is unstable since stability is related to the encirclements of the sum $C_m(s)P_{mu}(s) + C_m(s)P_{mu}(s)$.

2. In the QFT approach, performance specifications are translated to frequency-parameterized bounds in the Nichols chart. In tuning the controller, the loop frequency responses must lie within the regions defined by these bounds. In the design of the feedforward controller $C_m(s)$, this region can be very small. Due to the iterative nature of the QFT tuning step, the design of $L_m(s)$ in one iteration affects the performance bound on $L_e(s)$ in the subsequent step and so on. During tuning, it is possible that bounds at new frequencies have to be added to the loop-shaping environment if analysis of the closed-loop indicates unacceptable response at some frequencies not included in the original frequency set.

3. These bounds helped us to distribute the loads of bandwidth between the controllers as well as in making tradeoffs between performance and controller order. Figures 6-7 show the loop-shaping environments for the feedforward loop $L_m(j\omega)$ and the feedback loop $L_e(j\omega)$ with their respective bounds at $\omega = 547\ Hz$. Only a very small modification in the magnitude and phase of the feedforward loop $L_m(j2\pi547)$ is needed for the open-loop plot to lie within its bound (and hence meet performance), while a rather large magnitude increase in the feedback loop $L_e(j2\pi547)$ is needed to satisfy its bound. In general, it was observed that as $L_e(s)$ came closer to satisfying its bounds, the the resulting bounds on $L_m(s)$ become less stringent.

4. Performance is more easily achieved through the feedforward controller $C_m(s)$ when compared to the feedback controller $C_e(s)$. This is due to the existence of right half-plane zeroes in the ANC dynamic $P_{eu}(s)$ which precludes excessive gain in the feedback loop $L_e(s)$. On the other hand, since high gain generally results in improved robustness, the feedforward filter $C_m(s)$ had less effect in providing robustness against process variations when compared to the feedback controller. This appears to be the prominent benefit of feedback in this control architecture.

5. Another advantage in using a feedforward/feedback scheme over either a feedback-only or feedforward-only strategy is that it allows the designer to distribute the "bandwidth load" across $C_e(s)$ and $C_m(s)$. That is, for performance's sake, a certain level of "loop gain" is required, and this gain can be realized by distribution over both $C_m(s)$ and $C_e(s)$. The advantage to this is that neither $C_m(s)$ or $C_e(s)$ must bear the brunt of possibly destabilizng high gains and large roll-off rates which are necessary to avoid interaction with omnipresent high-frequency uncertainty. The magnitudes of these two controllers are shown in Figure 8.

The Experiment

The resulting controller was implemented in real-time to verify predicted performance. Figure 9 gives a schematic of the hardware setup along with the software configurations. The duct was 4" PVC and was 1.15 m long with one end open and a speaker located at the other end to simulate disturbances. The control speaker, detection microphone and error microphone were located 0.38 meters, 0.51 m and 0.21 m, respectively, from the open end. The hardware included: Radio Shack control speaker and the microphones, home-made microphone phantom power and pre-amp, a QSC power amplifier, and a Spectrum TMS320C30 DSP board housed in a PC. Data processing and analysis was done using an HP and a Siglab Analyzers.

The implementation of the (24th order) controller involved the following steps: (1) conversion of each controller to second-order sections, (2) discretization using the step-invariant method, (3) confirmation of discrete-time stability using the QFT Toolbox z-domain loop-shaping environment, and (4) download to the TMS320C30 DSP system board. Figure 10 shows the open-loop, simulated closed-loop and experimental closed-loop transfer functions from the disturbance $d(j\omega)$ to the error $e(j\omega)$. Note that similar attenuation levels were observed for both pure tones, swept sine and band-limited white noise.

Conclusions

Preliminary experiments indicate that good disturbance attenuation levels, which are competitive with adaptive schemes, can be achieved in an ANC system with an LTI control design approach. The design approach blends H_∞ synthesis and QFT design of feedforward and feedback controllers. First, an H_∞ design is performed on the nominal identified model with uncertainty due to identification error. Subsequently, the QFT environment is used with the experimental duct frequency response to improve performance and reduce controller complexity.

Present work is focused on further improvements by considering design trade-offs between the feedback and the feedforward controllers which are transparent within the QFT environment. Future work will consider design robustness to environmental and geometric variations in the duct.

References

[1] Fuller, C.R. and von Flowtow, A.H. (1995). "Active Control of Sound and Vibration", *Control Systems Magazine*, Vol. 15(6), pp. 9-19.

[2] Mehta, P.G., Zheng, Y., Hollot, C.V., and Chait, Y. (1996). "Active Noise Control in Ducts: Feedforward/Feedback Design by Blending H_∞ and QFT Methods", *Proceedings of 13th IFAC World Congress*, San Francisco.

[3] Nelson, P.A.and Elliot, S.J. (1992). *Active Control of Sound*, Academic Press, London.

[4] Widrow, B., Glover, J.R., McCool, J.M., Kaunitz, J., Williams, C.S., Hern, R.H., Zeidler, J.R., Dong, E. and Goodlin, R.C. (1975). "Adaptive Noise Canceling: Principles and Applications", *Procs. IEEE*, Vol. 63, pp. 1692-1716.

[5] Jiang, F., Ojiro, N., Ohmori, H. and Sano, A. (1995). "Adaptive Active Noise ControlSchemes in Time-Domain and transform-domains", *Procs. CDC*, pp. 2165-2172.

[6] Mehta, P.G. (1996). *Active Noise Control in One Dimensional Ducts : Fixed-Filter Designs*, M.S. dissertation, University of Massachusetts, Amherst.

[7] Adcock, J.L. (1987). "Curve Fitter for Pole-Zero Analysis", *Hewlett-Packard Journal*, pp. 33-36.

[8] Balas, G.J. and Doyle, J.C. (1990). "Identification and Robust Control forFlexible Structures", *IEEE Control Systems Magazine*, Vol. 10(4), pp. 51-58.

[9] Chiang, R.Y. and Safonov, M.G. (1992). *Robust Control Toolbox*, The MathWorks, Natick, MA.

[10] Borghesani, C., Chait, C. and Yaniv, O. (1995). *The Quantitative Feedback Theory Toolbox*, The MathWorks, Natick, MA.