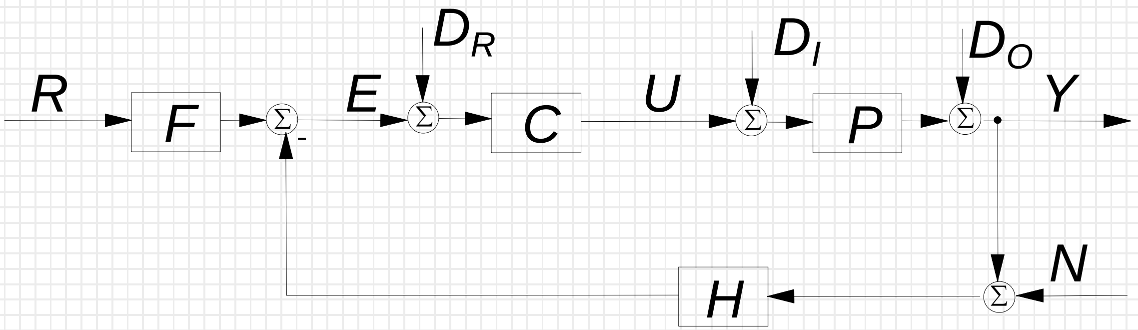
9. Two-Degrees-of-Freedom Design

In some feedback schemes we have additional degrees-of-freedom outside the feedback path. For example, feed forwarding known disturbance signals or reference signals. In this chapter we focus on an instance commonly referred to as a pre-filter (or input shaping) as shown below.



The controller C can be designed to shape S or T, but not both independently since

If the plant has large uncertainty, then large $|C|$ may be required over a large frequency range. This implies (F = 1):

also over a larger frequency range. If the desired speed of response matches this high bandwidth, then we are doing fine. If it is slower than what is expected, we're stuck with too high bandwidth and consequently "higher" power requirement.

Since we have already seen the disadvantage of "large" control bandwidth, it is suggested to remove this burden from C by "speeding up" the reference signal.

This point is illustrated below for the sensitivity reduction problem in Chapter 8 where we had an integrator plant with gain uncertainty

$$P = \frac{k}{s}, \quad k \in [1,3]$$

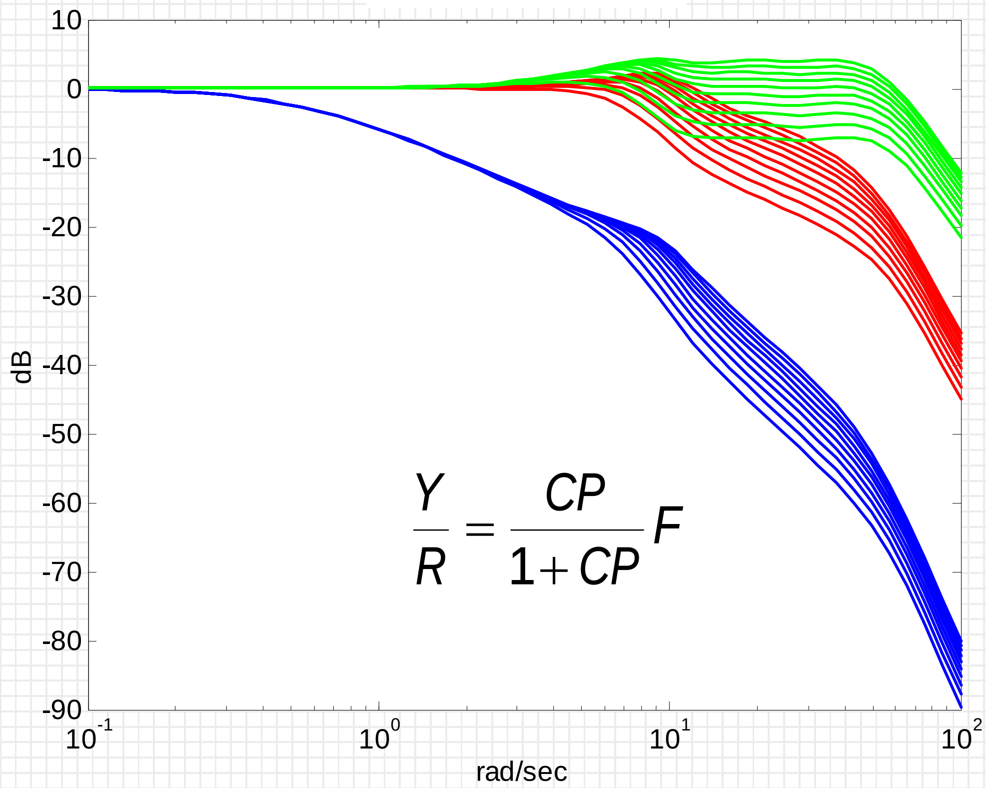
and the performance weight was

$$\left|\frac{1}{1+CP}\right| \le \min\left(\frac{\omega}{5}, 1.4\right), \quad \text{for all } \; P \in \mathcal{P}.$$

The design yielded $Y/R = T = S - 1$ with an an approximate bandwidth of 10 rad/sec. Per the earlier "cost of feedback" discussion, larger bandwidth increases the system's sensitivity to high frequency noise and uncertainties beyond what the specs call for w/o an appreciable transients improvement.

Note that Y/R depends also on the pre-filter F. Since it is OUTSIDE the feedback loop it cannot affect stability or closed-loop sensitivity. Once the controller C is designed, we can design F to modify speed of tracking response.

For example, using the same C, and two different pre-filters we can either increase or decrease tracking bandwidth as shown below.

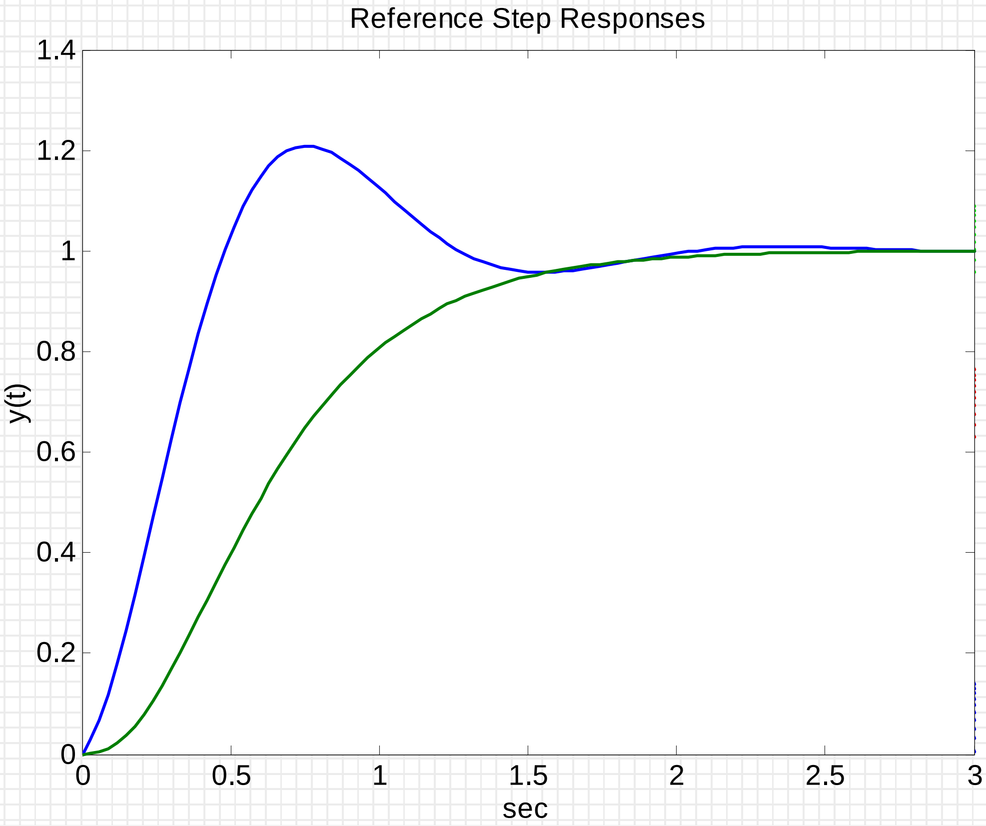


In summary, we conclude that the role of F is to shape the input tracking energy but not affect closed-loop robustness properties.

9.1. The QFT Tracking Problem

In QFT, the 2 DOF problem is historically motivated due to time-response considerations. Consider this problem: given an uncertain plant set, design a controller *C* and a pre-filter *F* such that the closed-loop step responses have maximal 2% settling time of 2 sec. and no larger than 20% overshoot.

These specs are depicted graphically below where the responses are to stay within the envelop.



The time-domain to frequency domain map is not one-to-one. However, for a large class of systems we can achieve reasonable mapping. The idea is to assume a dominant second-order model (or third etc') and find damping ratios and natural frequencies that result in above step responses. An excellent discussion on this procedure (also for disturbance rejection models) can be found in the QFT references (IH's book). Two such models are:

The conjecture is that if we have robust stability and

then

The feedback design problem can now be stated. Design a robustly stabilizing controller and a pre-filter such that

$$\left|T_l(j\omega)\right| \le \left|\frac{CP}{1+CP}F(j\omega)\right| \le \left|T_u(j\omega)\right|, \quad \text{for all } P \in \mathcal{P}.$$

The design of C and F can be elegantly decoupled as discussed earlier. We first design C such that closed-loop variations are within specs taking advantage of the derivations below. Taking log gives

Since the pre-filter is fixed

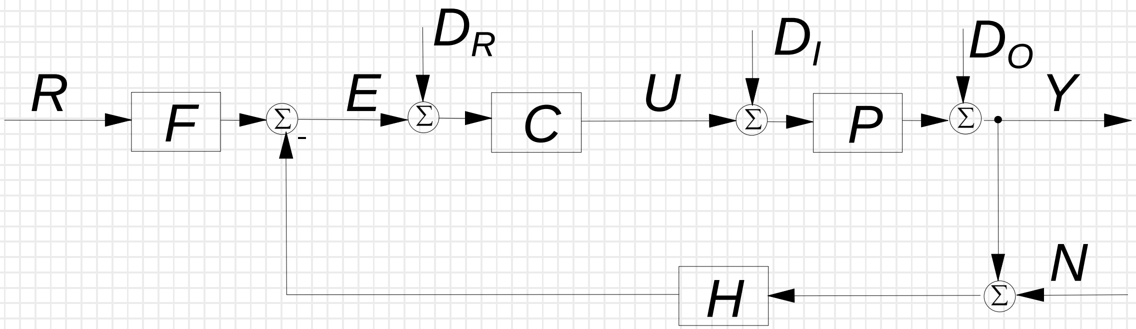
We try first to design C such that

then construct a stable, minimum-phase F such that its magnitude satisfies (always feasible)

Let us now go through such a QFT design using an example.

9.2. A 2 DOF QFT Example

Consider a standard feedback structure shown below (example 2 in the *QFT Toolbox*)



where

$$\mathcal{P} = \left\{ P(s) = \frac{ka}{s(s+a)}, \quad k \in [1,10], a \in [1,10] \right\}.$$

Design C and F to achieve robust stability, robust margins

$$\left| \frac{CP}{1+CP}(j\omega) \right| \leq 1.2, \quad \text{for all } P \in \mathcal{P}$$

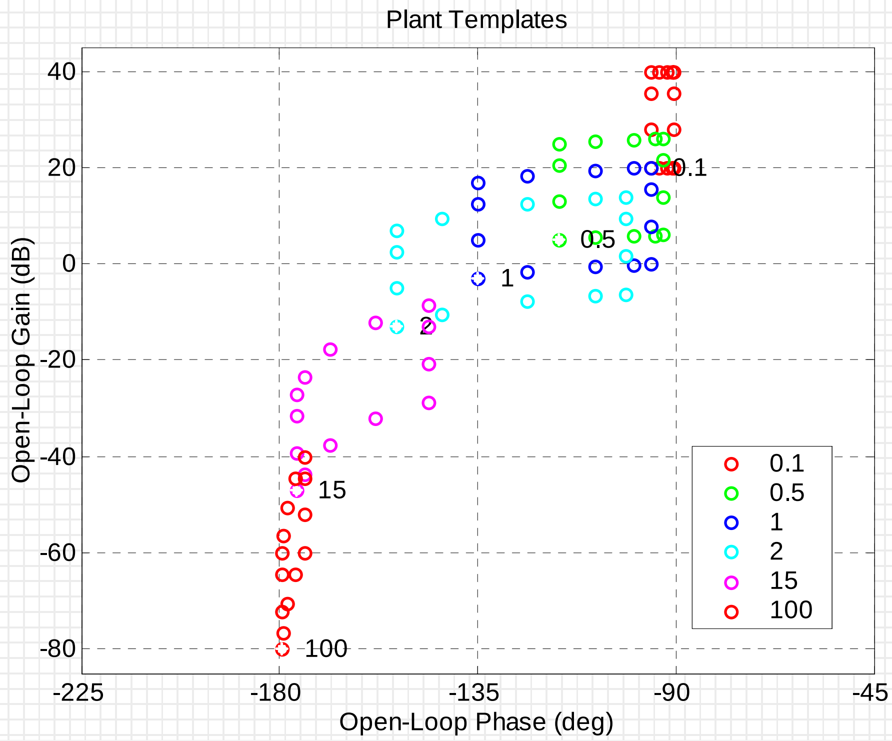
and robust tracking

$$\left|T_l(j\omega)\right| \le \left|\frac{CP}{1+CP}F(j\omega)\right| \le \left|T_u(j\omega)\right|, \quad \text{for all } P \in \mathcal{P}.$$

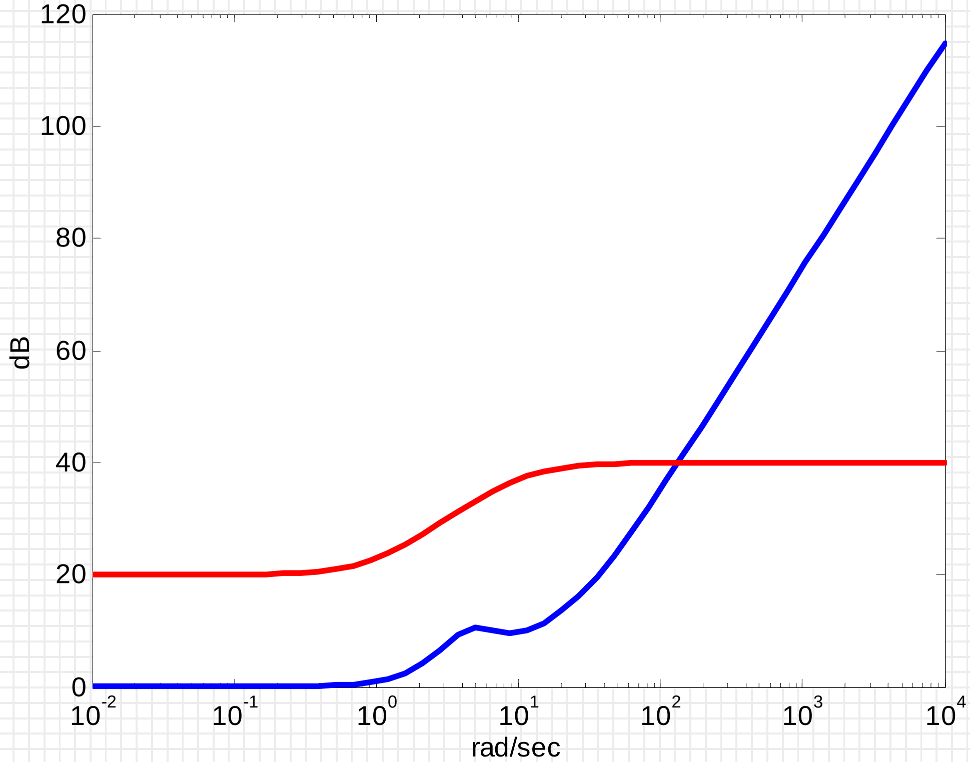
where

$$T_u = \frac{0.6854s + 20}{s^2 + 4s + 20} \qquad T_l = \frac{120}{(s+3)(s+4)(s+10)}.$$

We start by evaluating plant templates to understand their low, mid and high frequency behavior.



We now verify that the tracking specs are consistent with plant uncertainty. Specifically, that the desired closed-loop variations are actually smaller than open-loop variations. If not, there's no need for feedback (ch9_chk_specs.m).

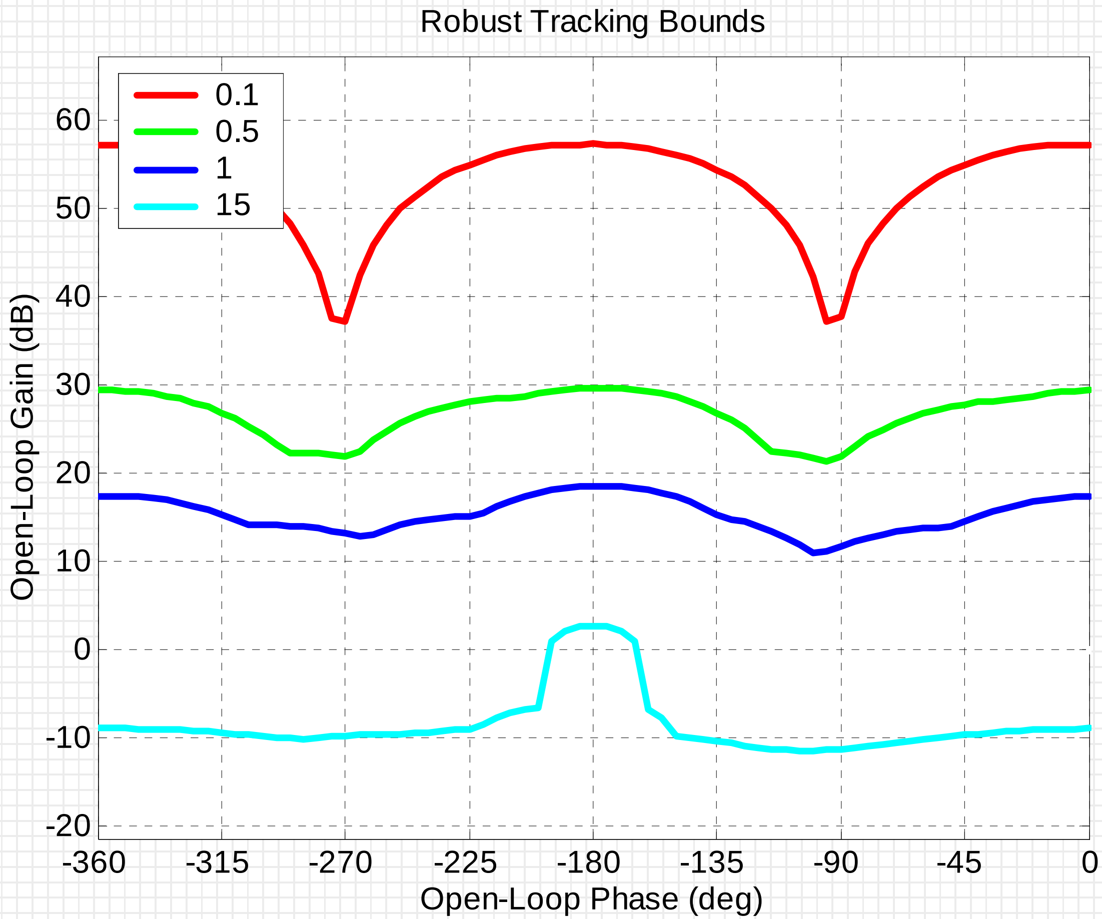


Beyond ≈100 we no longer need feedback to reduce closed-loop variations. It should be consistent with desired bandwidth for speed of response. If necessary, augment T_u and T_l with high-frequency poles and zeros.

The design process itself is similar to what we have already seen in earlier chapters. Complete details can be found in `ch9_qftex2.m`. The tracking bounds can be computed using

```
wbd7 = w(1:5);
mu = tf(0.6584*[1,30],[1,4,19.752]);
ml = tf(120,[1,17,82,120]);
W7 = [mu;ml]; % tracking weight
bdb7 = sisobnds(7,wbd7,W7,P);
```

Note that unlike any of the other performance problems, tracking bounds cannot be computed in closed form and therefore require more computational load.

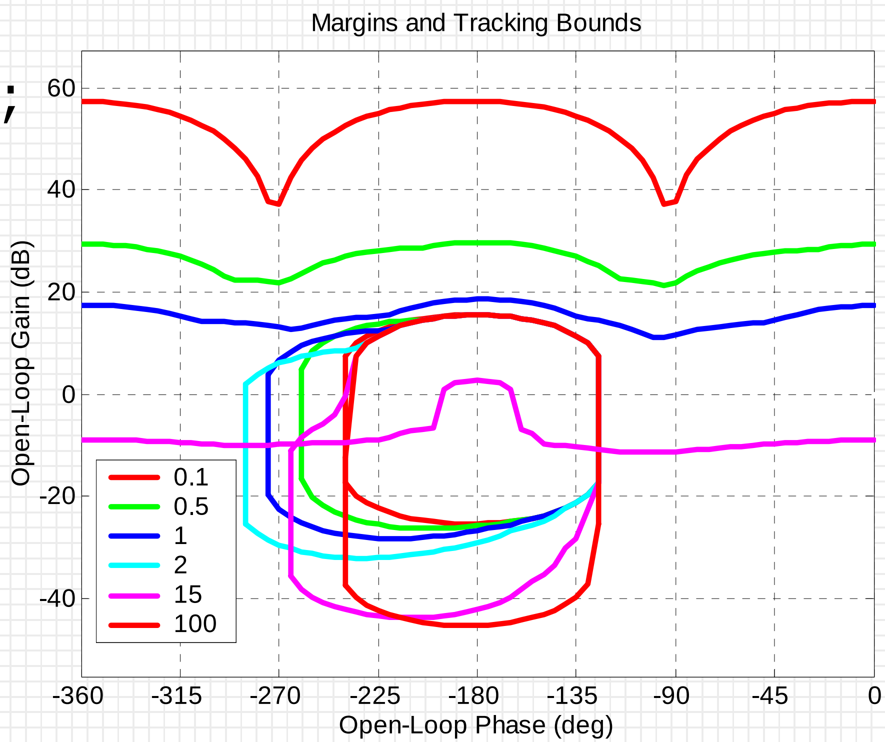


Also, at a fixed phase, tracking bounds can be multi-valued§. The Toolbox takes the max value for ease of manipulation (numerical studies indicate that the resulting conservatism is minimal).

§ Wang, G.G., Chen, C.W., and Wang, S.H., 1991, "Equation for loop bound in Quantitative Feedback Theory," *Procs. CDC*, pp. 2968-2969.

It is instructive to consider all bounds (margins and tracking) on the same graph.

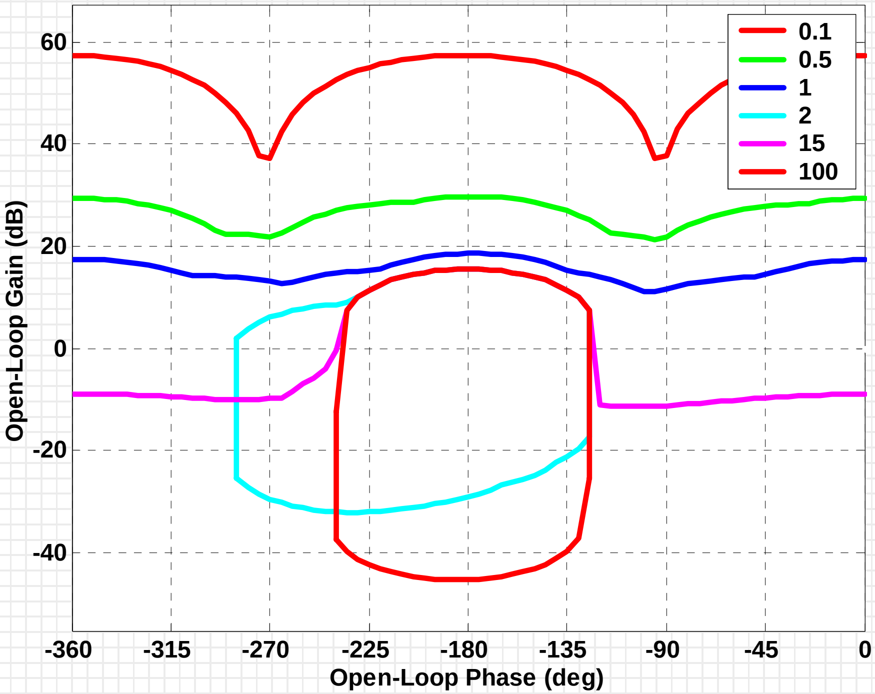
```
bdb=grpbnds(bdb1,bdb7);
plotbnds(bdb);
```



Notes:

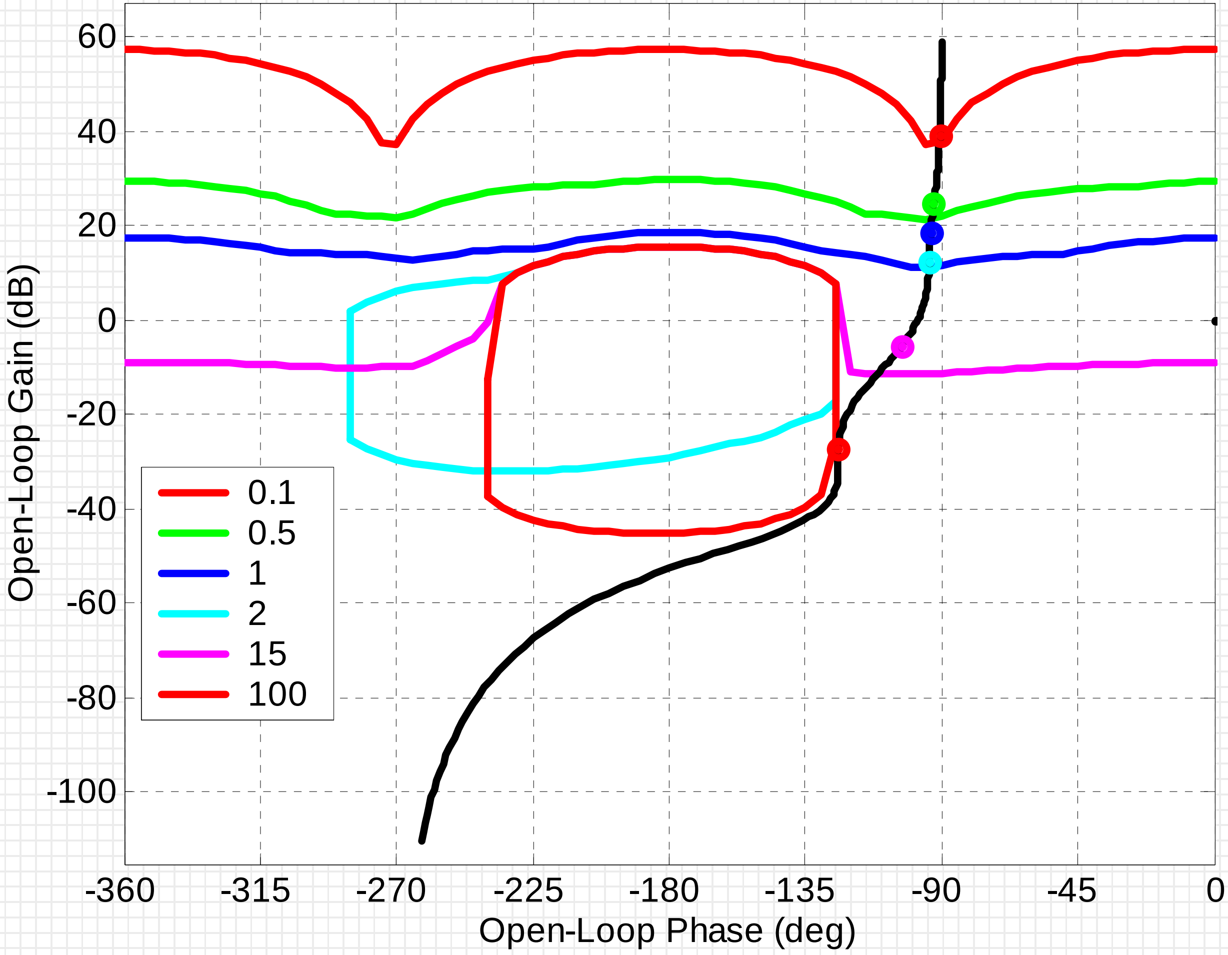
To simplify the presentation, we often loop shape only with the worse-case bound, i.e., their intersection.

```
ubdb=sectbnds(bdb);
plotbnds(ubdb);
```

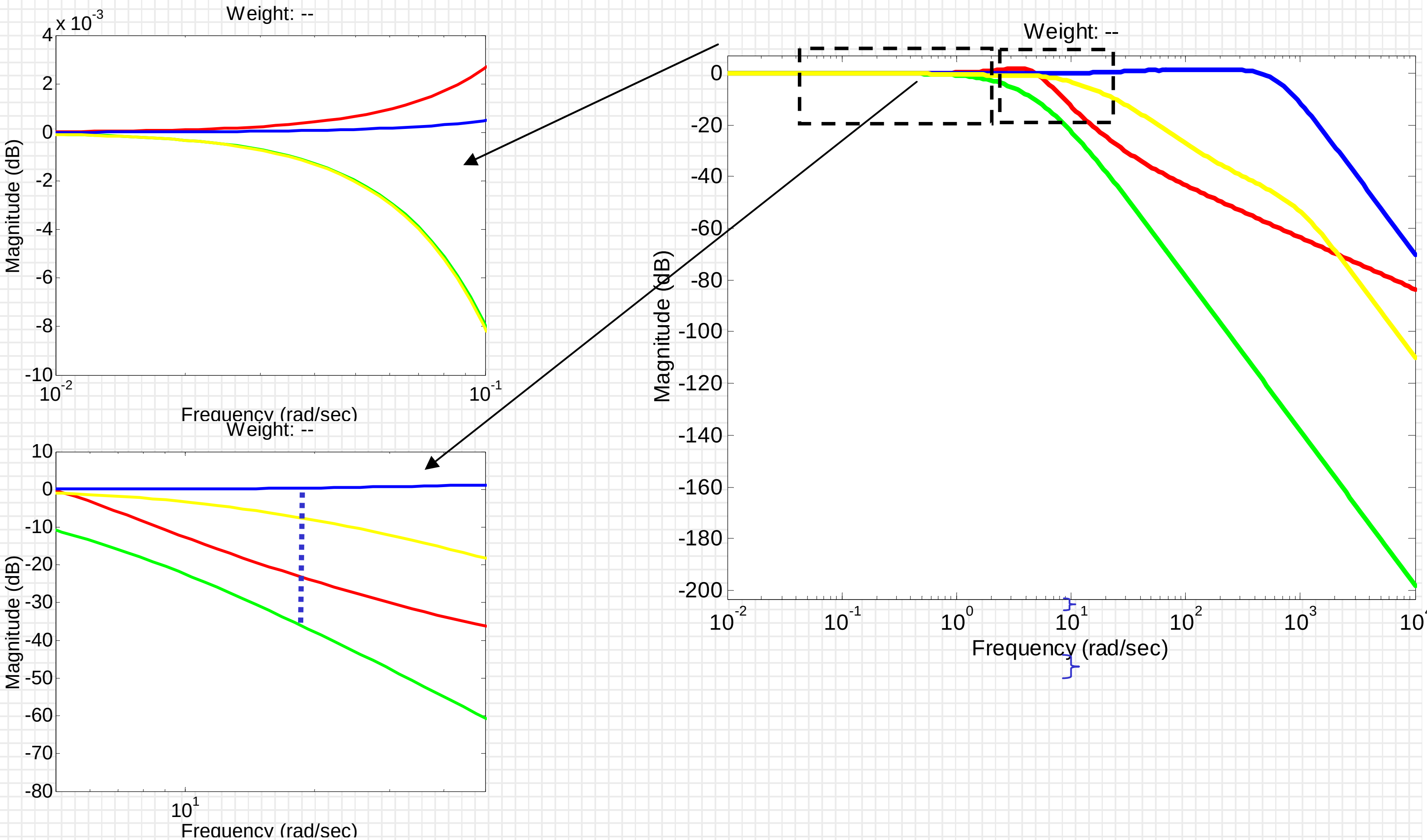


Note that at, for example ω = 15, the bounds show portions of both tracking and margin bounds.

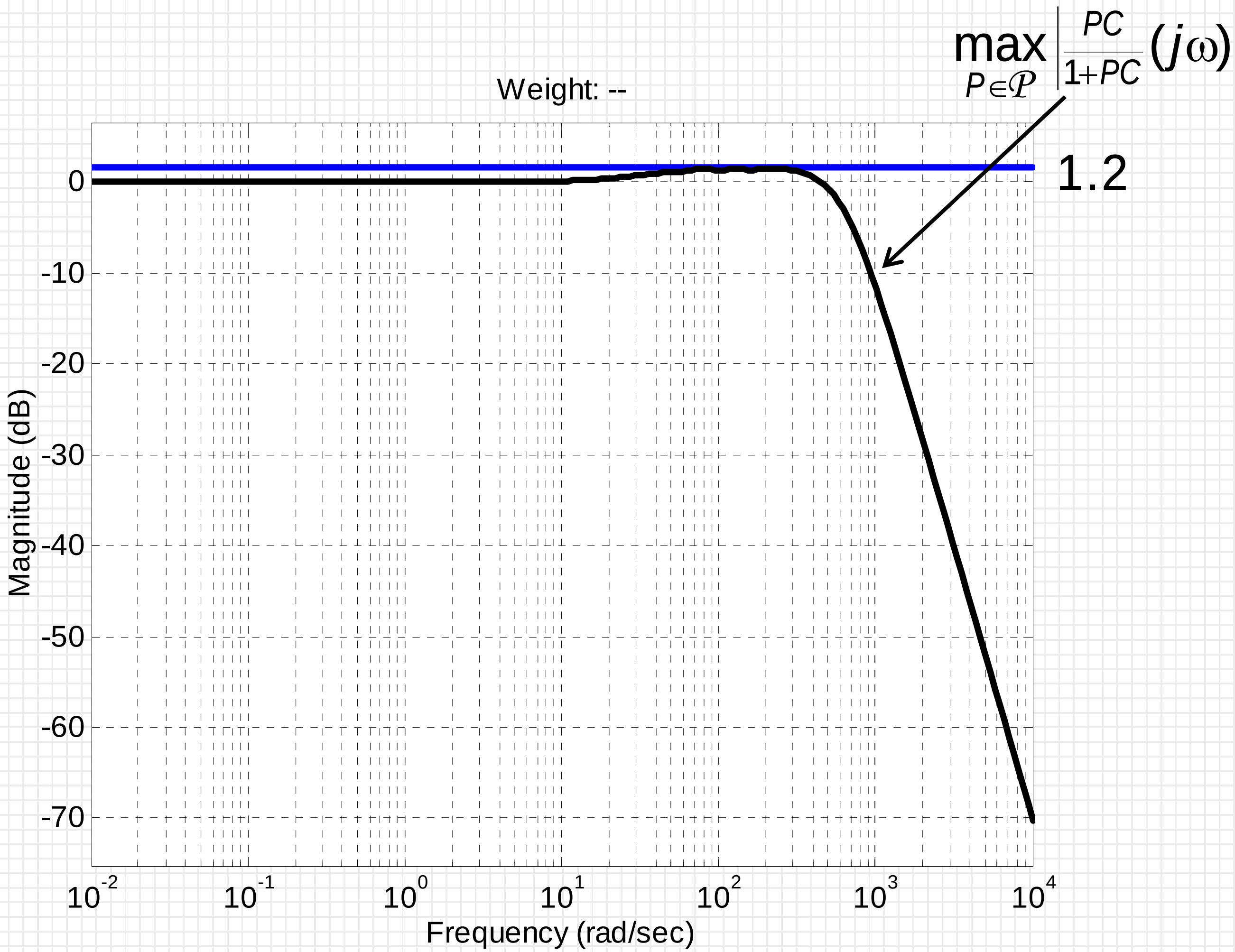
We are ready to loop shape (i.e., design the controller). We leave out the details of the 3rd order controller design for the M-file (`qftex2.m`). The resulting loop shaping figure looks like



As discussed earlier, the controller is designed to reduce closed-loop variations to within the specs, but not to place closed-loop tracking responses between $|T_u|$ and $|T_l|$. This is seen below.

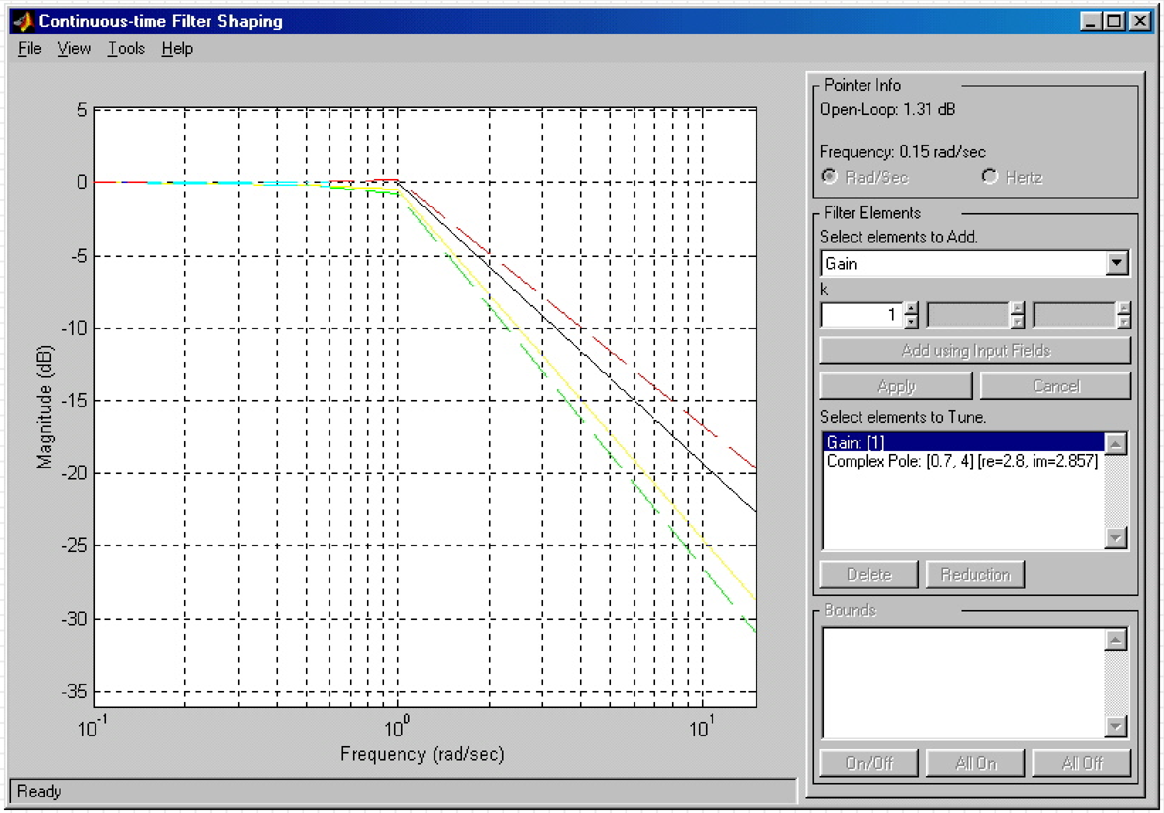


Evaluating the margin performance shows that we are meeting the spec (since the bounds were satisfied during loop shaping and the shape of the plant template is fixed at the high- freq range).

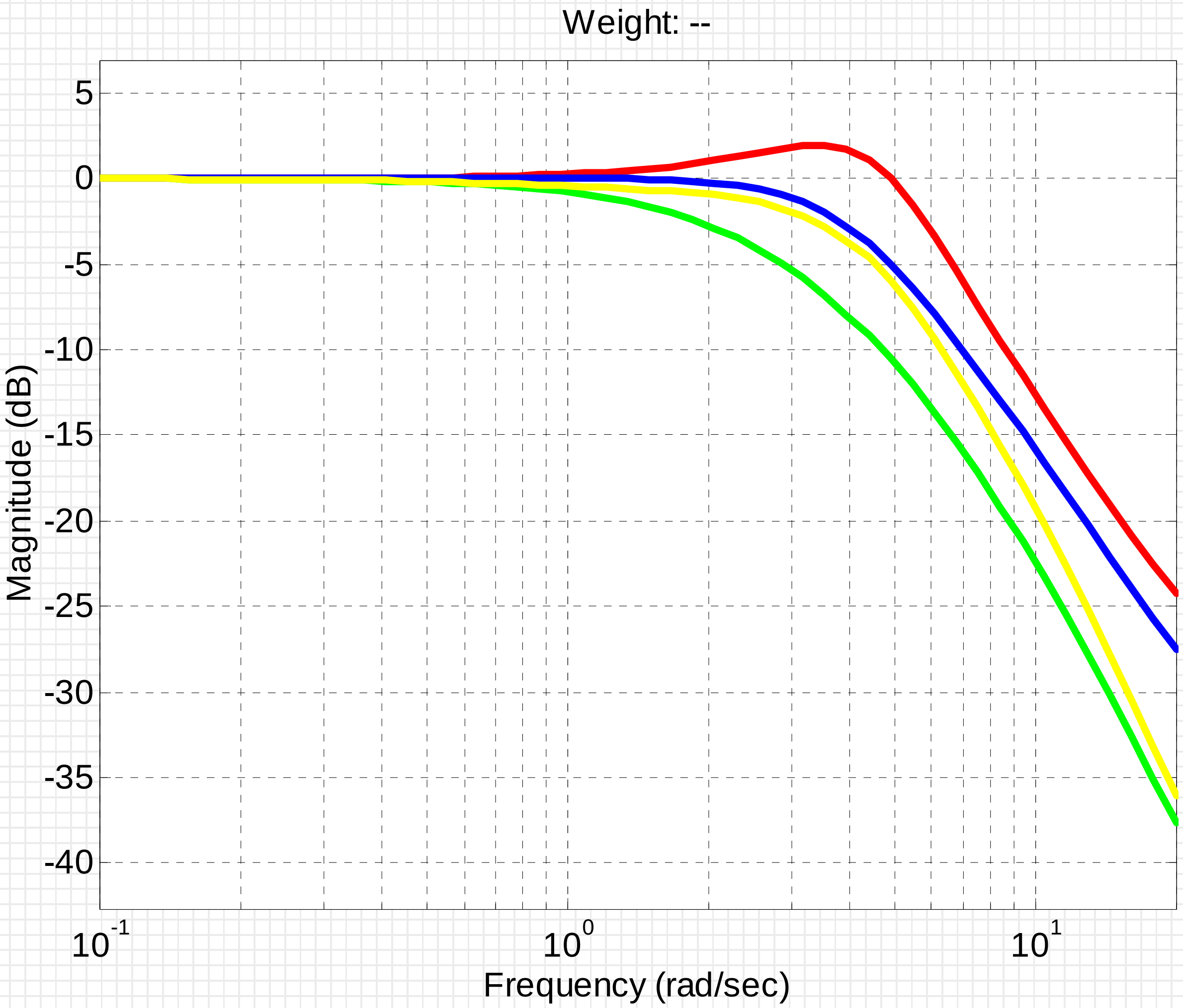


At this point we have successfully completed the design of *C* to meet margin specs and reduce closed-loop tracking variations. Now we are ready to complete the design. Pre-filter shaping can be done using a similar GUI used to loop shape the controller:

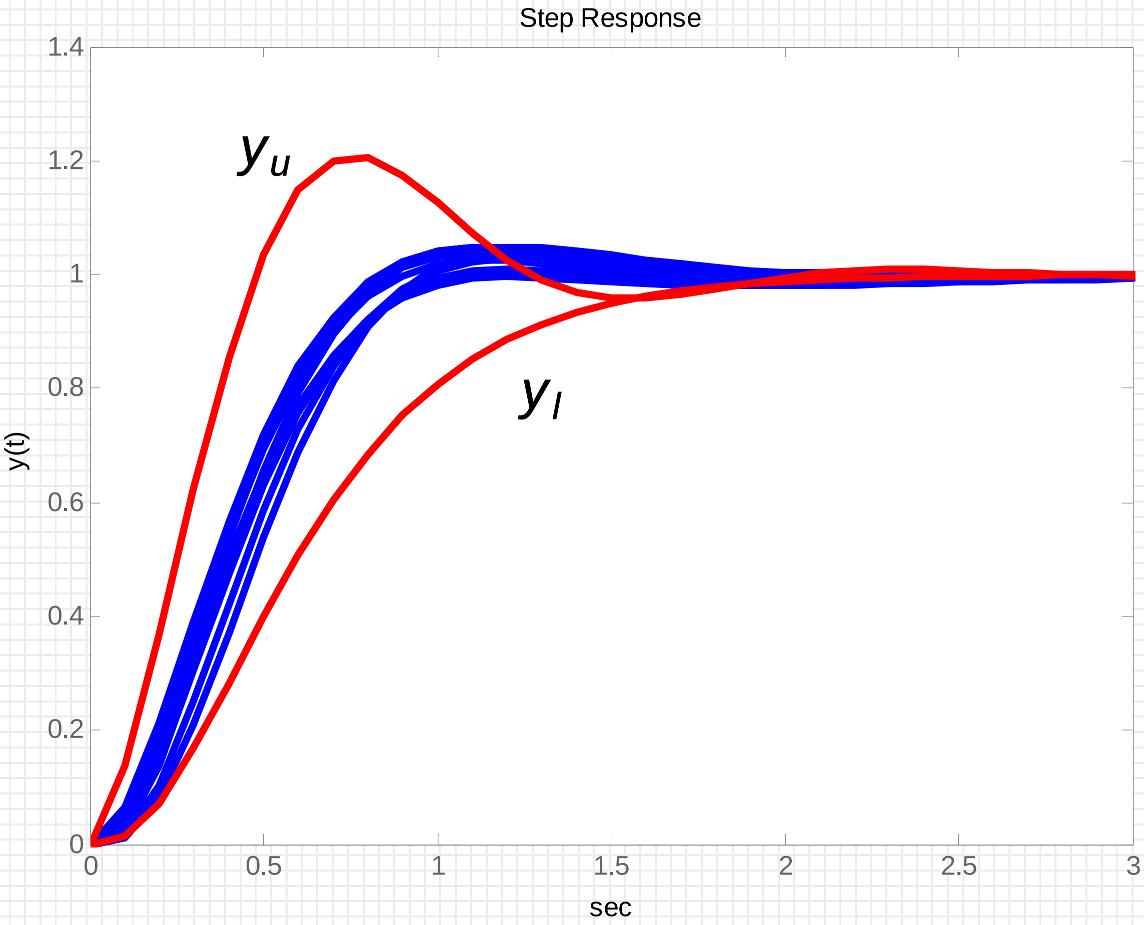
```
pfshape(7,w,W7,P,[],C);
```



Evaluating tracking performance gives the following.

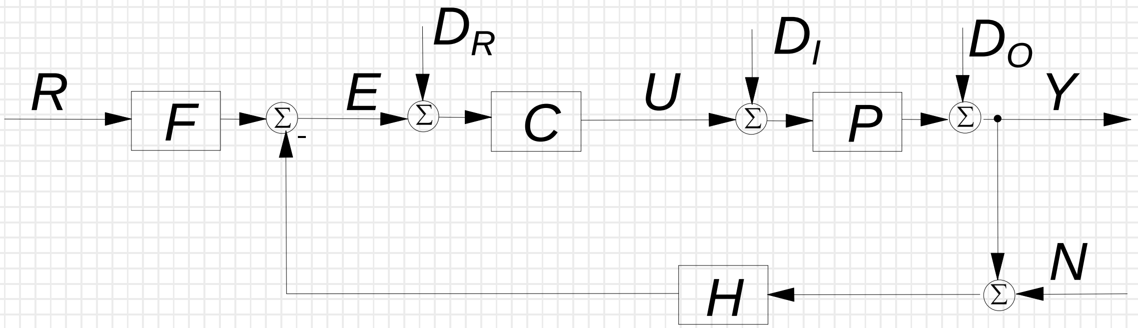


What about the time responses? While we realize the lack of 1-1 mapping, the resulting transients are close to meeting these specs too (`ch9.qftex2_tr.m`). Additional tuning of the pre-filter will possibly contain the responses within the envelop.



9.3. Notes

A tighter formulation of the 2 DOF tracking problem is model matching where the pre-filter is a given nominal tracking function F_m.



In this scenario, the problem is to design C to achieve

If we select $F = F_m$, the above reduces to a sensitivity reduction problem

Note that the 2 DOF tracking problem should result in a lower bandwidth controller than the corresponding model matching problem turned sensitivity reduction problem. There is no reason to believe that the choice of nominal desired function F_m is best from input-output standpoint due to the uncertainties.

An alternative method for translating tracking response specs from time-domain into frequency-domain is given in the reference below. The idea is to use energy-type performance criterion of the form

where *y* denoted closed-loop tracking response, and *m* and *v* are specified time functions.

Krishnan, K.R., Cruickshanks, A, 1977, “Frequency domain design of feedback systems for specified insensitivity of time domain response to parameter variations,” *IJC*, Vol. 25(4), pp. 609-620.

A weaker version of the above is given by

which essentially means we are trying to match energies, not exact values of the responses. The advantage is that there's 1-1 mapping in terms of energy from time to frequency domains.

The original tracking specs are converted into this formulation as follows. Let $y_u(t)$ and $y_l(t)$ denote the upper and lower responses defining the performance envelop. Define

The equivalent frequency-domain representation becomes

and if we conveniently select

we end up with a sensitivity reduction problem

This formulation can lead to design conservatism, but given the difficulty in time to frequency domain mapping, it seems a reasonable alternative.

9.4. Homework

1. Prove that if

$$\log\Delta\left|P(j\omega)\right| \le \log\left|T_u(j\omega)\right| - \log\left|T_l(j\omega)\right|$$

then the tracking problem can solved with a pre-filter without feedback.

2. Given the plant

$$\mathcal{P} = \left\{ P(s) = \frac{k\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}, \quad k\in[2,6],\ \omega_n^2=5,\ \zeta\in[.3,.45] \right\},$$

design a feedback system that robustly stabilizes the plant, meets the margin spec

$$\left|\frac{1}{1+CP}\right| \le 2, \quad \omega \ge 0, \quad P \in \mathcal{P},$$

meets these tracking specs for a step reference input

- 20% overshoot, and
- 2% settling time of 2 sec.

with minimal “cost of feedback”.

