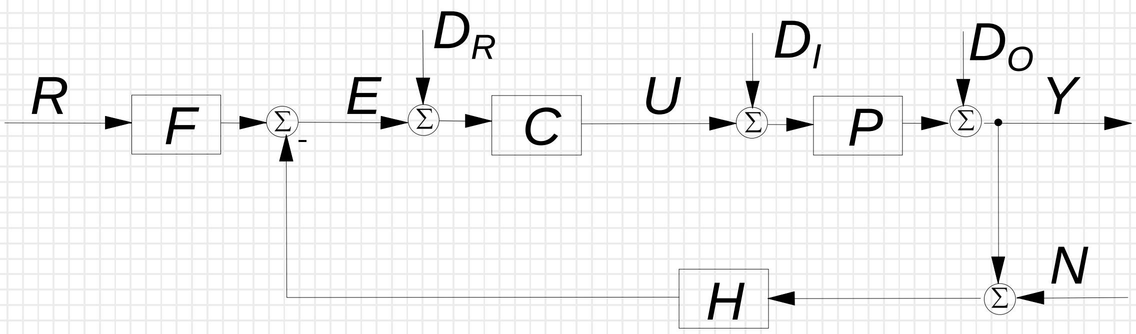
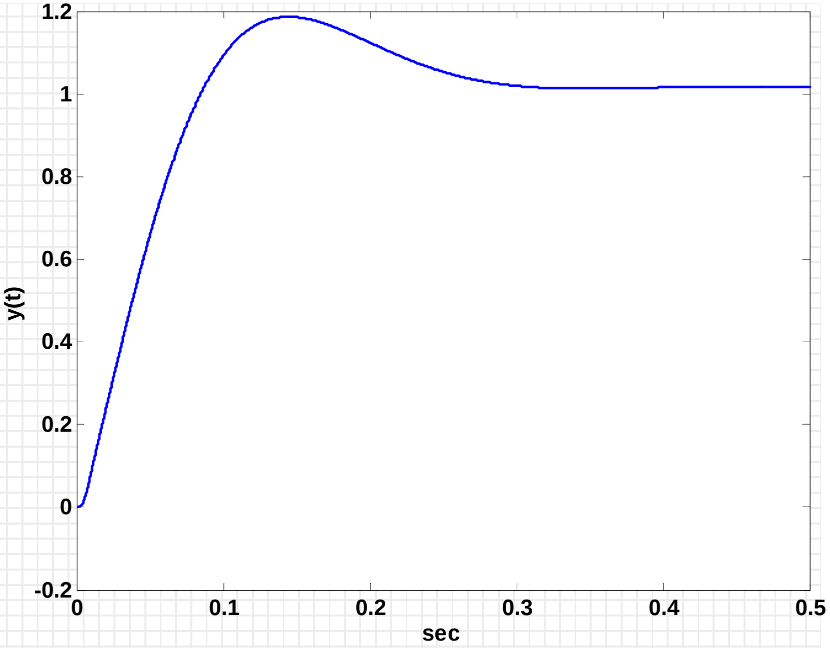
8. One-Degree-of-Freedom Design

In Chapter 7 we completed a QFT design to achieve robust stability with specific robust stability margins. Let us now expand this design procedure to a more general single-loop feedback setting.

We begin by considering various 1 DOF problems for the generic block diagram shown below (assume F = 1).

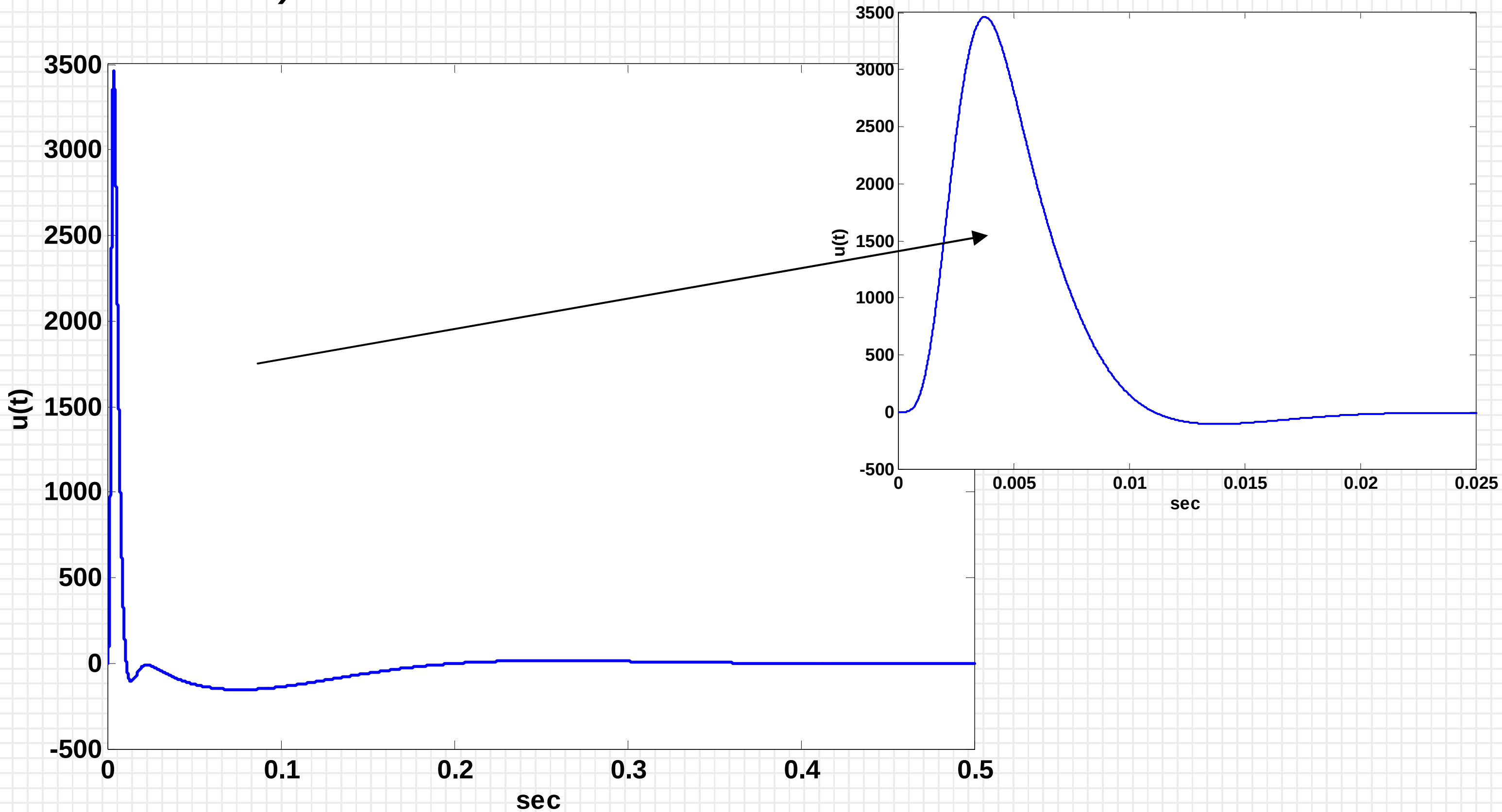


A typical constraint in many applications is in terms of the control effort *U* when limited actuation power calls for a spec in the form $|u(t)| \leq u_{max}$. The step response looks fne and "harmless", but real applications must consider power requirement.



This time-domain constraint cannot be easily converted into a frequency-domain spec without some design conservatism. At any rate, a frequency-domain spec on the control effort due to reference commands has this form ($H = F = 1$):

How do we construct a reasonable weight $W_u(w)$? Here's one approach. Let us consider once again the example in Chapter 7 with the "better" design. The control effort for a reference step command is shown below for the nominal plant and "best" design from last chapter `ch7_highorder_red.shp` (`ch8_u.mdl`).



8.1. Control Effort Minimization

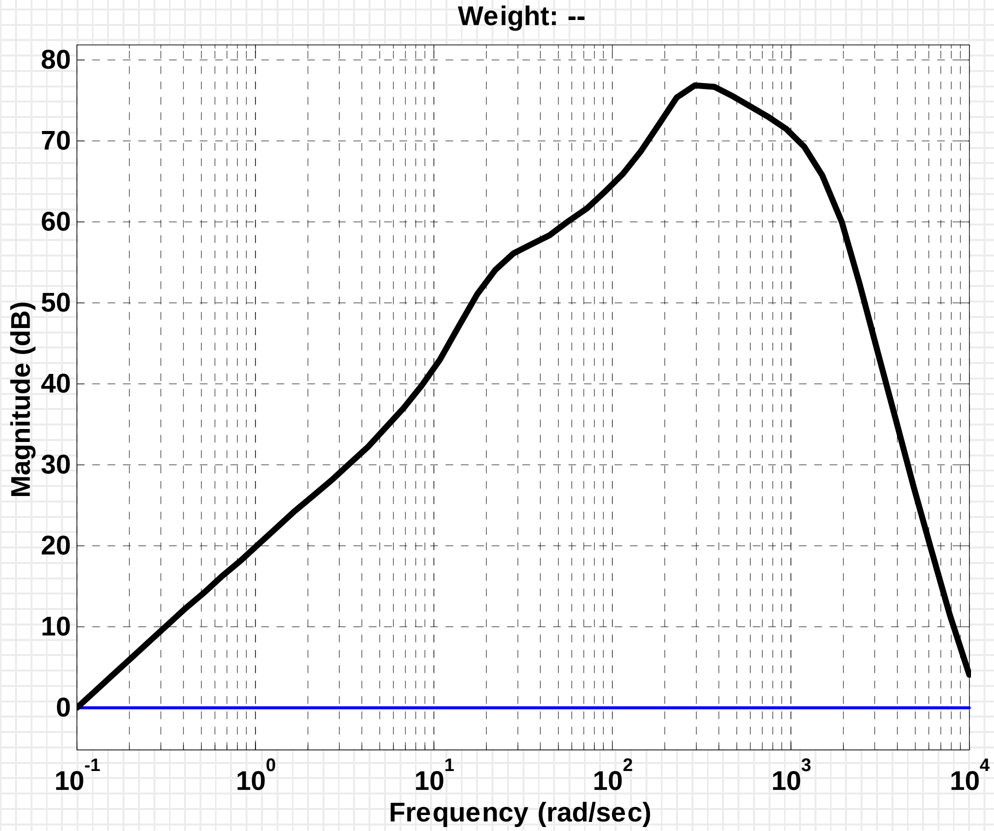
Let us first evaluate the control effort dynamics in the frequency response. We use `ch8_a_u.m` which consists of the following key command:

```
chksiso(4,w,1,P,[],C);
```

Another way to compute the above is:

```
tu = feedback(C,P); % or C/(1+C*P);
tufr = squeeze(freqresp(tu,w));
mtufr = abs(tufr);
z = max(mtufr,[],2);
semilogx(w,20*log10(z));
```

The max amplitude at each frequency is shown below.

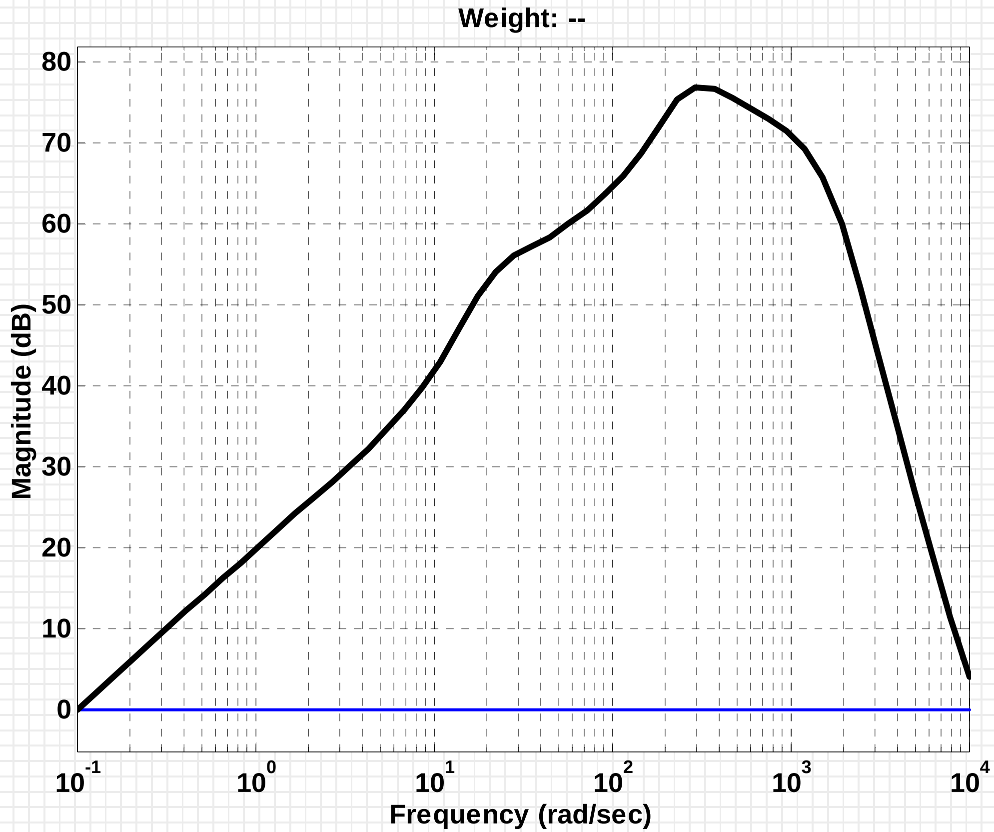


Reducing the DC gain is one way to cut down this huge peaking. But it also affects the whole frequency response. So we would like to avoid this drastic solution.

Inverse Laplace transform requires the whole frequency response to compute the time response

however, as a rule, we should limit large peaking in $|U/R|$.

In our example, say we attempt to cap peaking to below 1000 (60 dB), and try to maintain the same response elsewhere. In modern control, we would construct a weight, such as $W_u(w)$, such that:

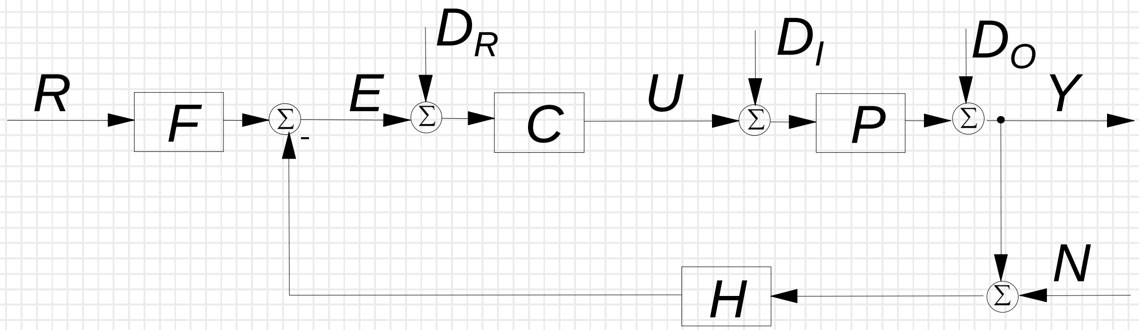


Rather then focusing on the entire frequency range, we concentrate on the range of interest $\omega \in [50,2000]$ and try to maintain similar $C(j\omega)$ elsewhere, especially in the lower end. Appropriate QFT bounds are computed using the function sisobnds.m in the same manner done for the margin-type problem earlier :

```
bdb = sisobnds(ptype,w,Ws,P,R,nom,...)
```

- bdb: a matrix with values of bound on nominal loop, in dB, at -5° spacing from 0° to -360° (default). *note: ASCII but in a special toolbox format.*
- ptype: specify the input/output problem(4 here)
- w: freq vector
- Ws: performance weight or spec
- P: LTI/FRD array or complex freq resp matrix
- R: multiplicative uncertainty
- nom: nominal plant index

Various input/output problem types are shown below.



ptype	Input/Output Problem	Example
1	$\left\lvert \frac{PCH}{1+PCH} \right\rvert \le Ws_1$	gain/phase margins
2	$\left\lvert \frac{1}{1+PCH} \right\rvert \le Ws_2$	sensitivity reduction
3	$\left\lvert \frac{P}{1+PCH} \right\rvert \le Ws_3$	input disturbance rejection
4	$\left\lvert \frac{C}{1+PCH} \right\rvert \le Ws_4$	control effort

ptype	Input/Output Problem	Example
5	$\left\lvert \frac{CH}{1+PCH} \right\rvert \le Ws_5$	control effort (w/sensor dynamics)
6	$\left\lvert \frac{PC}{1+PCH} \right\rvert \le Ws_6$	1 DOF tracking
7	$Ws_{7a} \le \left\lvert \frac{C}{1+PCH} \right\rvert \le Ws_{7b}$	2 DOF tracking
8	$\left\lvert \frac{H}{1+PCH} \right\rvert \le Ws_8$	output dist. rej. (w/sensor dynamics)
9	$\left\lvert \frac{PH}{1+PCH} \right\rvert \le Ws_9$	input dist. rej. (w/sensor dynamics)

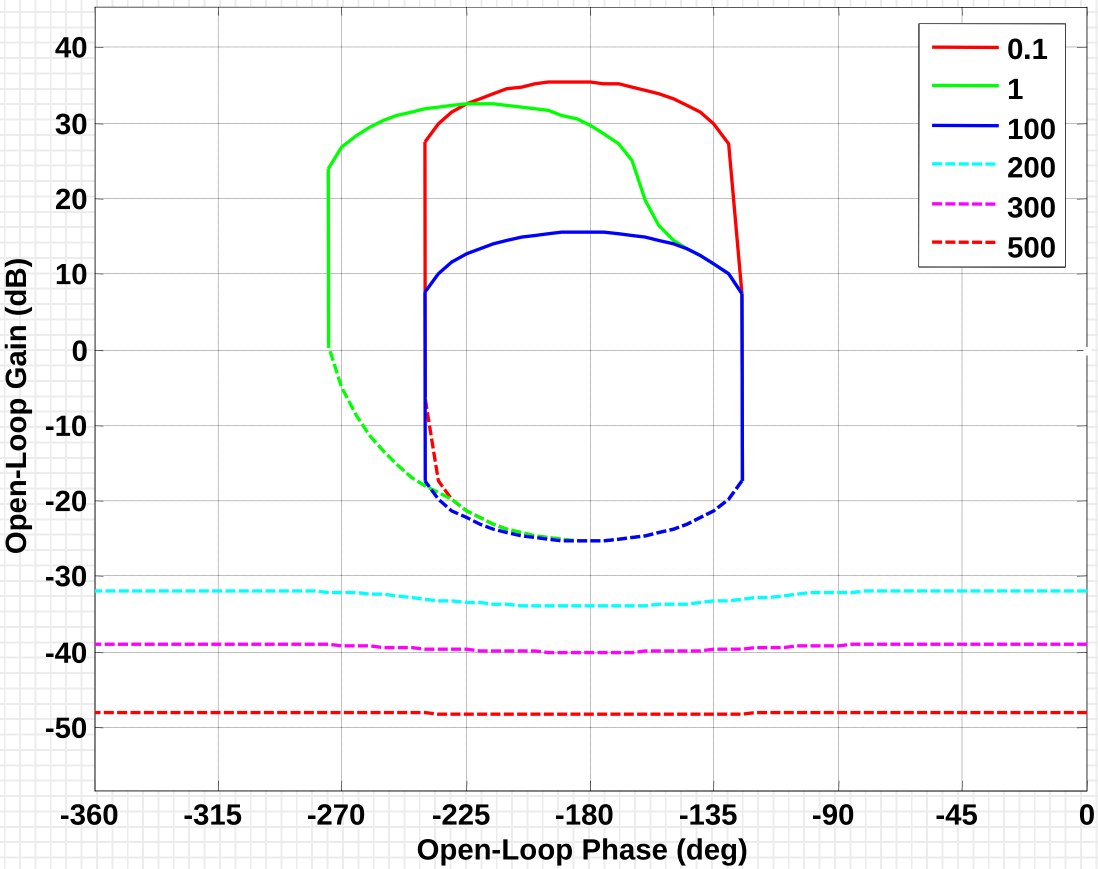
The QFT bounds are computed in `ch8_b.m` (over a smaller freq band):

```
wbd1 = [.1,1,100];
W1 = 1.2;  % margin weight
bdb1 = sisobnds(1,wbd1,W1,P);

wbd4 = [200,300,500];
W4 = 1000; % control effort weight
bdb4 = sisobnds(4,wbd4,W4,P);

bdb = grpbnds(bdb1,bdb4);
plotbnds(bdb)
```


The result is shown below.



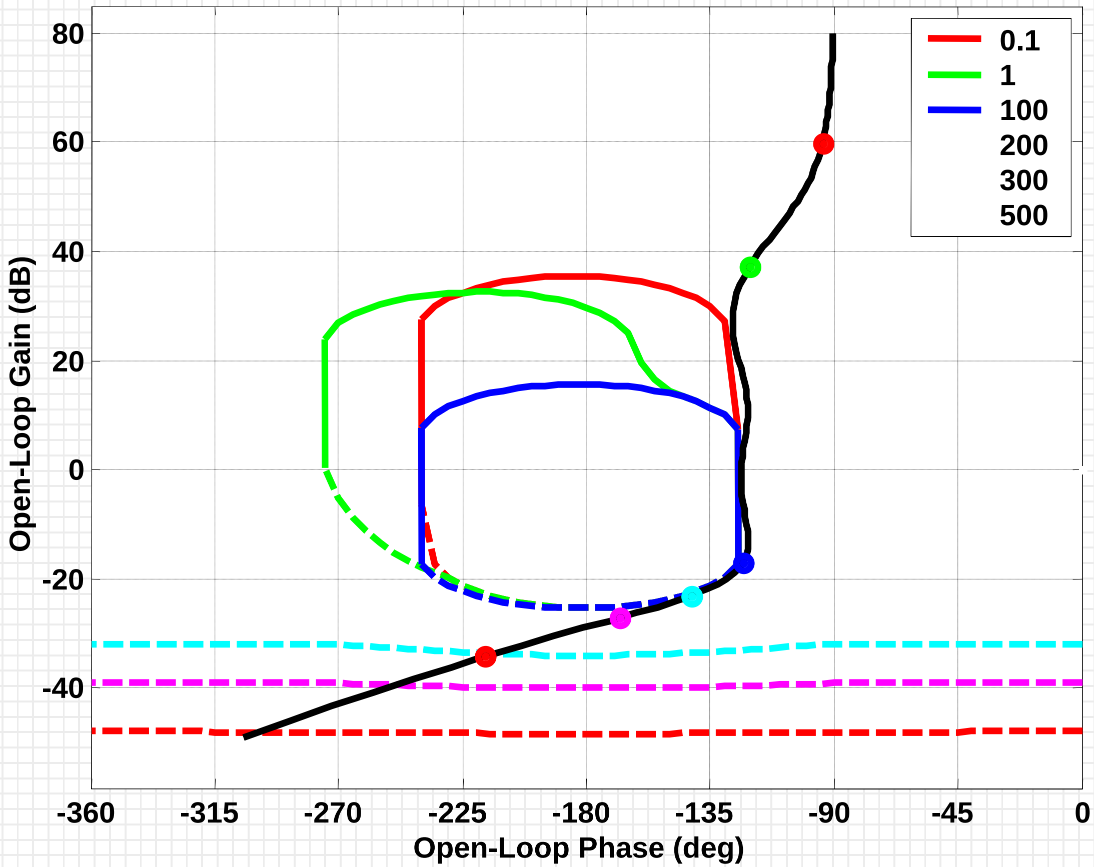
Note that a control effort constraint amounts to limiting the controller bandwidth. Specifically, at some point above crossover frequency ($|L|$=0 dB) the loop gain tend to decrease rapidly so

The bounds shown exhibit this relation (except that they are computed for $L_0=CP_0$ and not C). In this class of bandwidth-limiting problems, you should expect the bounds to lie below 0 dB and be plotted using dashed lines indicating that the loop must lie on or below them.

If the weight is sufficiently relaxed, then the only constraint would be that L_0 does not come near –1, i.e., $|1+L_0|>>0$ to avoid peaking in closed-loop transfer functions. This is a margin-type problem and you will see the "expanded" M-circle bounds as seen in the gain/phase margin problem.

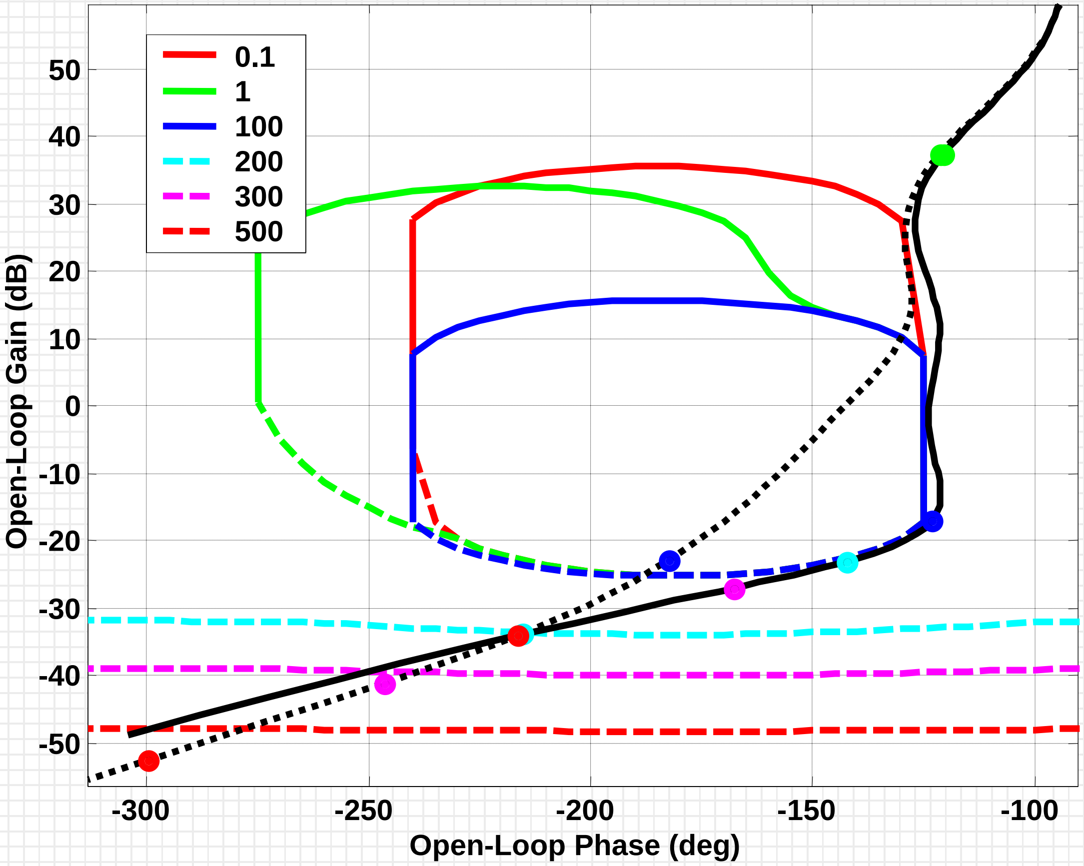
Also note that at the high-frequency range even with similar templates (e.g., vertical lines) and similar spec (60 dB), the corresponding bounds on L_0 are NOT similar (lack of symmetry).

Proceeding directly to loop shaping (`ch8_lp.m`), and retrieving the controller design for the margins spec (File|Open, then select `ch7_highorder_red.shp`) results in:

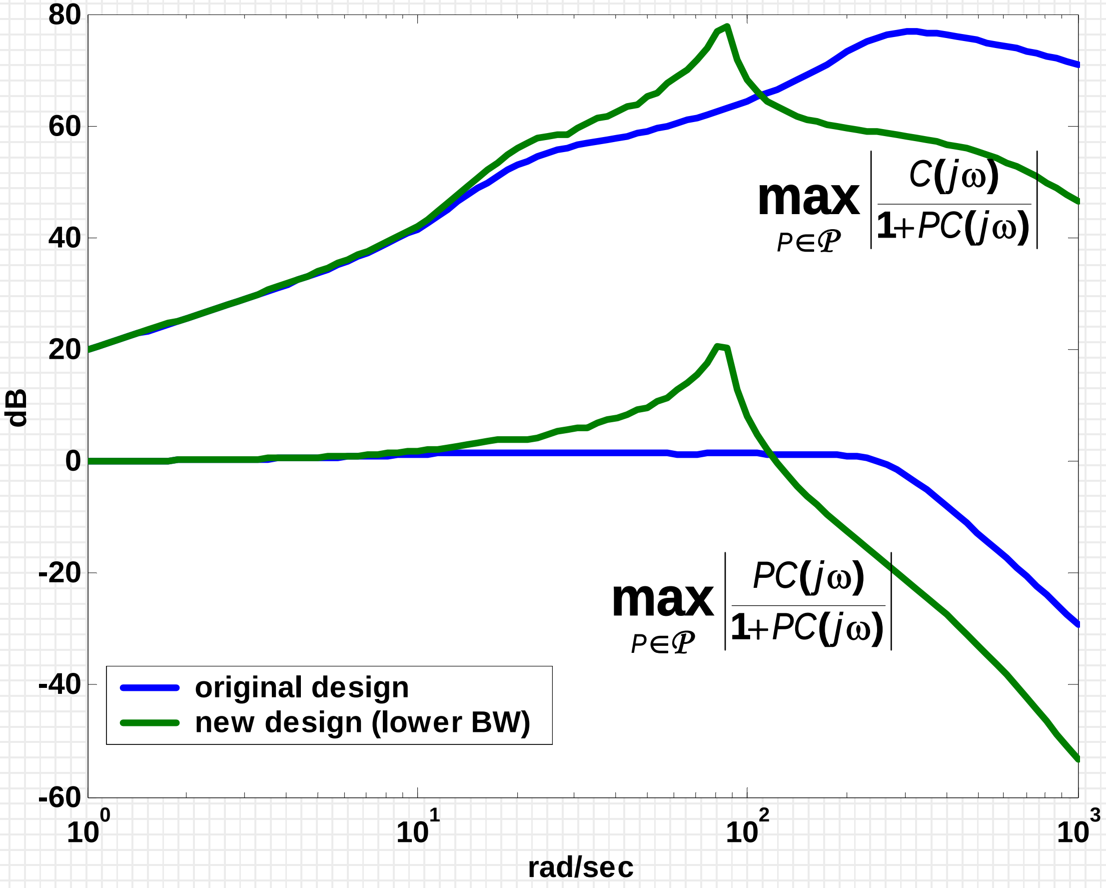


Can we reduce the loop gain in the range just above ω >100 without affecting the low and mid-freq dynamics?

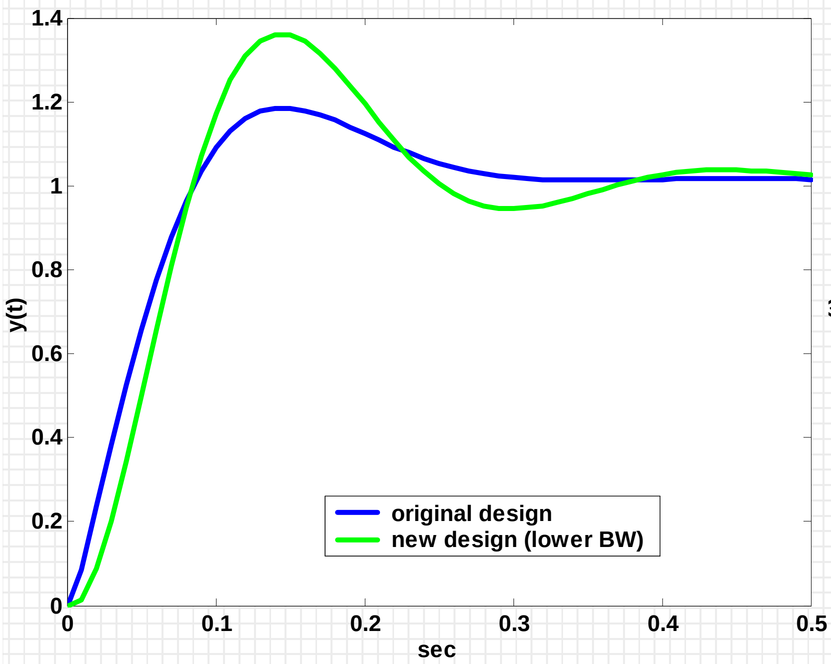
Probably not, unless we are willing to tradeoff bandwidth (and possibly steady-state gain). Loop gain reduction must be accompanied with commensurate phase lag. Adding low-pass dynamics in that range illustrates this inevitable tradeoff (`ch8_newdesign.shp`).

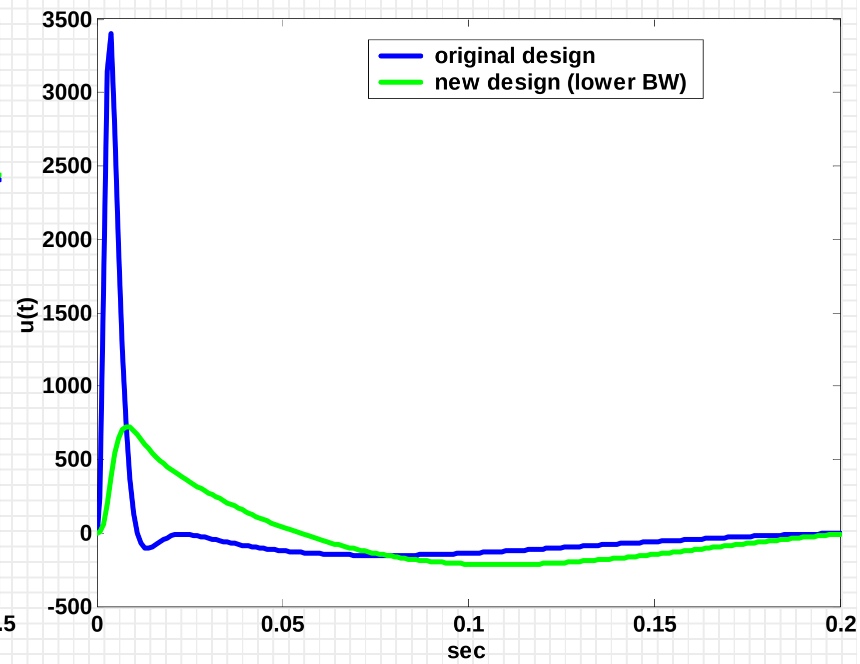


It is instructive to compare the closed-loop frequency responses of both the control efforts and tracking relations as shown below.



The nominal reference step responses below show that reduction in the control effort peak has been achieved by satisfying the bounds without steady-state change, of course, at the cost of increased overshoot (smaller margins) and somewhat slower transients (lower BW).





8.2. Sensitivity Reduction

Another common specification involve the sensitivity function. For example, in a CD-ROM the laser beam points on the spinning disk. Reading encoded information reliably can occur only if most of this spot lies on top of the desired track. This constraint on the error signal E leads to a sensitivity reduction problem of the form:

Rejection of disturbance entering the loop at the plant output, e.g., DC offsets, also gives rise to a sensitivity reduction form (if $H = 1$):

In a sensitivity reduction problem we distinguish between a W_S<1 case and a margin (or peaking constraint) weight where W_S>1. The former weight requires high loop gains since to achieve

we need

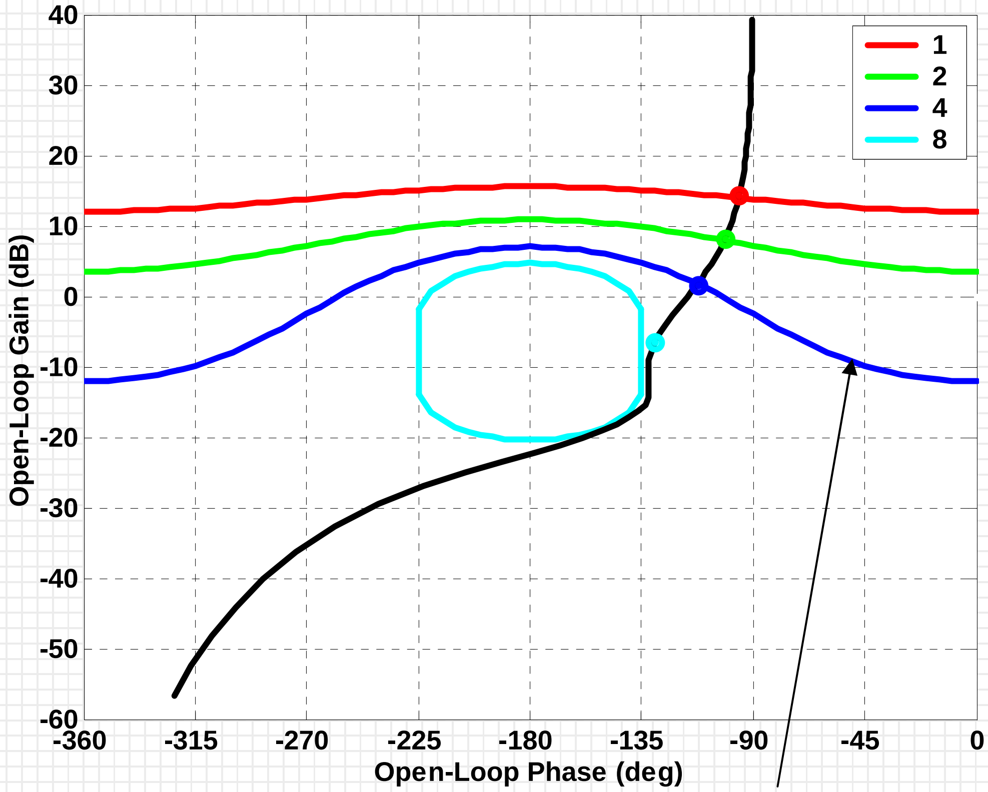
The later weight only constrains the loop around the critical point so to avoid peaking, but places no strict high gain or bandwidth constraint on the loop since $C = 0$ readily solves the problem.

Let us illustrate this point using a simple example.

Example. Consider an integrator plant with gain uncertainty

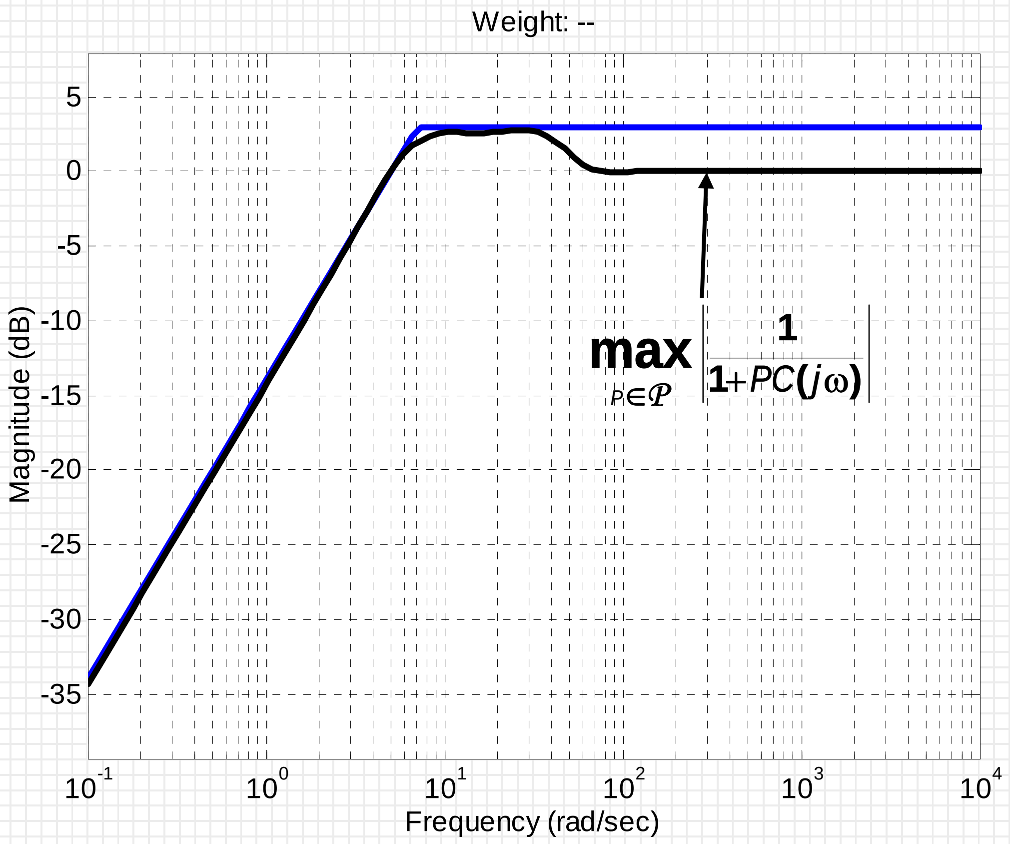
It is required that the sensitivity function be less then $\omega/5$ at low frequencies and never exceed 1.4. The weight has this form

The steps in a QFT design process should be automatic by now: study plant uncertainty via templates, compute bounds at several frequencies in the range of interest, then loop shaping. The details of such design can be found in `ch8_example.m`. A 3rd order controller (`ch8_example.shp`) resulted in the following design.



Note: the bound at ω = 4 requires loop gains near 0 dB. With $|L| << 0$ dB, $T \to L$ and , $S \to 1$, so there's little benefit from feedback and there's no need to maintain the high loop gain beyond the ω = 4 range. However, we must avoid peaking which is why margin bounds are the only type used at high frequencies. More on that later.

Evaluating the closed-loop response (`ch8_a_ex.m`) shows that, as expected, since the loop lies right on its bound at the low frequencies, monotonicity of both the plant and the spec guaranteed satisfaction of the weight at even lower (or higher) frequencies.



Theorem: Assume $L(s)$ has at least a 2-pole roll-off and N unstable poles. Then,

We observe above that at the low-freq range, where $|S| << 1$ ("benefits of feedback"), the integral is negative. So for an open-loop stable system, we must have $|S| > 1$ at some frequency band. Since peaking in closed-loop response is not desired, can we push the peaking in $|S|$ to very high frequencies where it would have far less effect?

Bode, 1945, Freudenberg and Looze, 1985.

Freudenberg and Looze, 1985, also show that if

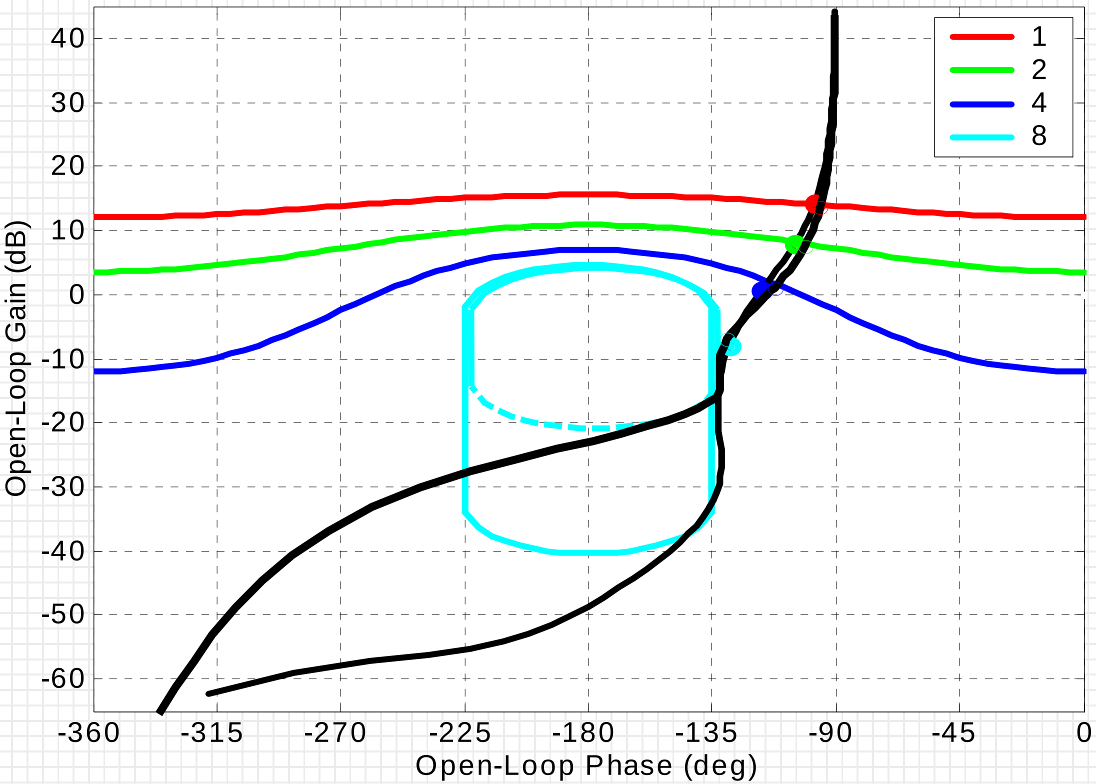
then

Hence $|S|>1$ must occur at $\omega < \omega_c$ if ε is very small. This is clearly seen in our example.

8.3. Notes

In the previous example we have glanced over a crucial aspect of feedback, namely the effect of high-frequency uncertainty on the "cost of feedback". For this purpose, consider the same plant with 10 times gain uncertainty

Taking the same performance spec, we know that the "low" frequency bounds remain unchanged since the lowest plant gain is the one dictating the required minimal loop gain. Both plants have same lowest gain of $k = 1$. A QFT procedure results in the following design (`ch8_example_b.m`, `ch8_example_b.shp`).



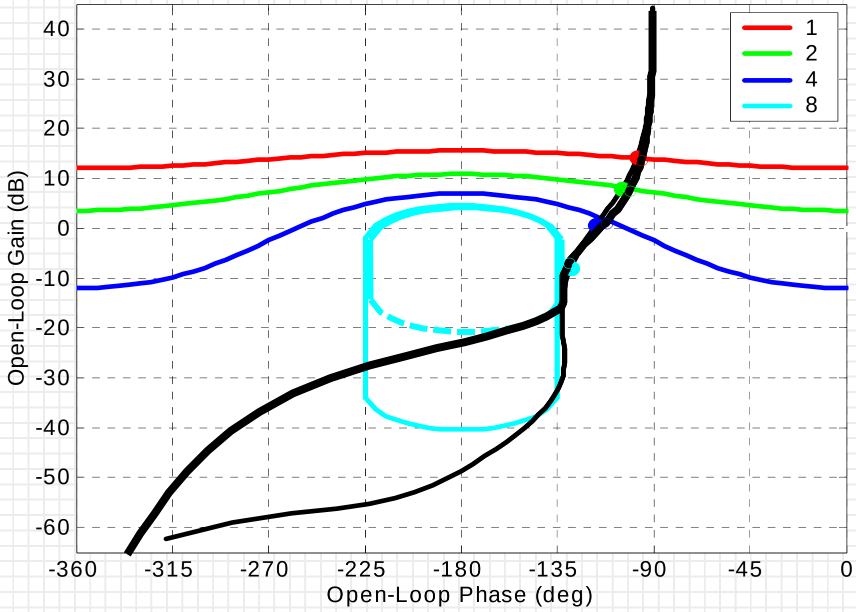
Both loops have similar dynamics until about $\omega = 8$. However, the increased gain uncertainty prevents the new loop from "turning" the corner at $\omega = 15$. The margin bound (or high-frequency bound) is 20 dB longer at the bottom forcing the new loop to maintain phase of no worse than –135° (the right edge of the margin bound).

The end result is that larger gain uncertainty forces higher control bandwidth. Yet this additional bandwidth is wasted since it occurs at a frequency band where $|L| < -20$ dB. Moreover, this larger bandwidth implies increases sensitivity to un-modeled dynamics in that range.

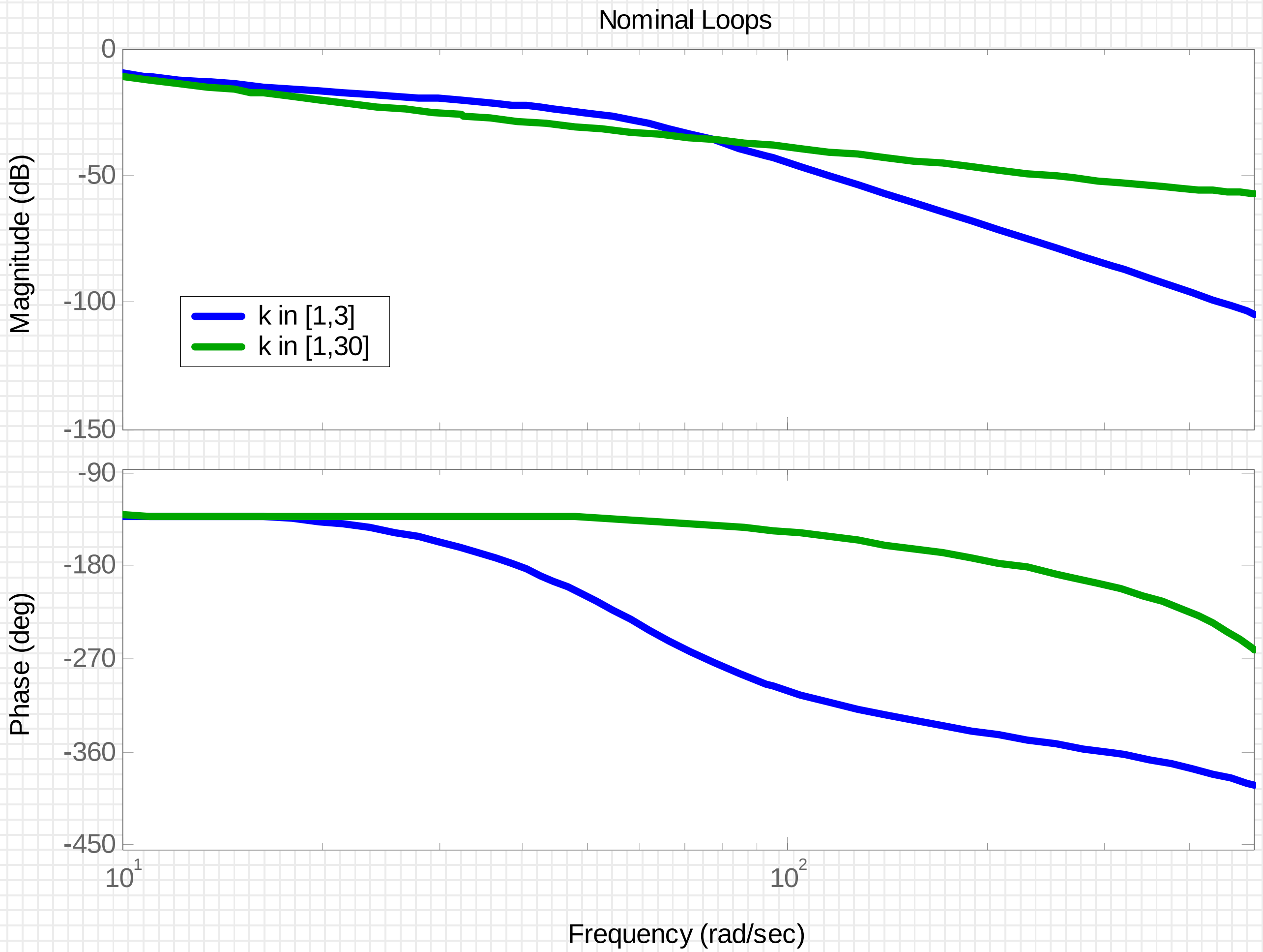
It is possible to estimate the “cost” of this larger gain uncertainty (i.e., larger margin bounds) in terms of loop bandwidth. From the well-known Bode’s gain-phase relation we know that given a stable and mp loop with $L(0) > 0$, its average slope in dB/decade over some frequency band can be determined from its average phase in that band:

So increased phase lag allows faster gain reduction of L (e.g., a 2^{nd}-order pole vs. a 1^{st}-order pole). Also, a similar relation holds for loops not satisfying the assumptions but with worse phase lag. Due to our margin bound, the loop can have at most –135° in this frequency range, implying a slope of

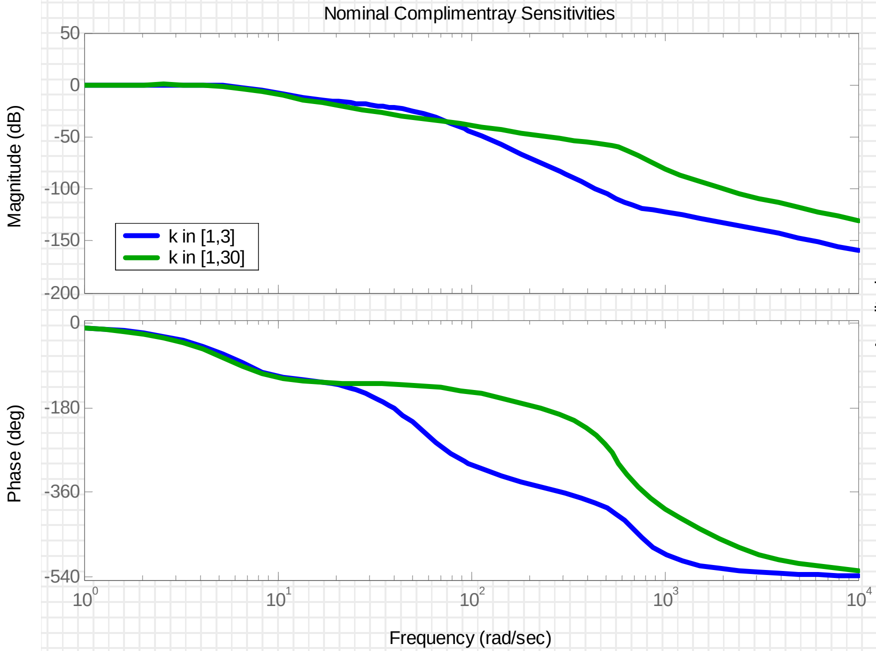
In our example, the top right-hand corner of the bound is at about 0 dB and both loops are there at approximately $\omega = 4$. The original loop can turn the corner at about –20 dB. It will need a bit over 2 octaves to drop from 0 to –20 dB at this constrained slope of –9 dB/oct: from 4 to about 16 rad/sec. Indeed this is seen in the actual loop response. On the other hand, the new loop will need about a decade plus one octave to accomplish the same thing: from 4 to about 80 rad/sec, a value confirmed by the actual loop dynamics.

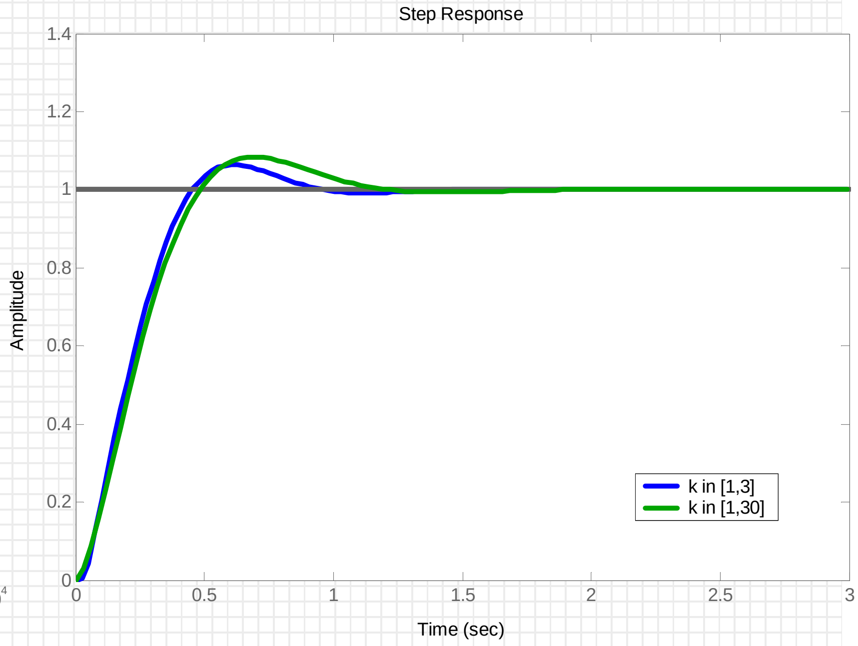


The mag plots of the two nominal loops illustrate this point.



The loop is required to maintain its gain over a larger frequency range without any input/output tracking benefits is clearly seen from the mag of the corresponding (nominal) complimentary sensitivity functions and step responses.





8.4. Homework

1. Explain why bounds on L_0 for similar specs and similar templates are also similar for sensitivity and complimentary sensitivity problems (assume H = 1). And why this is false for a control effort specification.

2. Consider an uncertain plant with gain uncertainty

$$\mathcal{P} = \left\{ P(s) = \frac{k}{s(s+1)} : \quad k \in [1,5] \right\}.$$

Design a controller $C(s)$ that (i) robustly stabilizes the system, (ii) achieves $|S| \leq [0.05, 0.2, 0.8]$ at ω = [1,2,4] rad/sec.,(iii) satisfies robust margins of the form $|S| \leq 3.5$ dB at all ω, and (iv) minimizes the high-frequency loop gain.

Next, consider the same specs and a similar plant with smaller gain uncertainty

$$\mathcal{P} = \left\{ P(s) = \frac{k}{s(s+1)} : \quad k \in [1,2] \right\}.$$

Design a new controller to take advantage of the reduced uncertainty. Compare the frequency responses of the two controllers and sensitivity and complimentary sensitivity functions. Considering reference step responses, is anything gained by having a larger bandwidth controller? Are there any disadvantages (hint: sensor noise and robustness to high-frequency un-modeled dynamics)? Illustrate these via simulations.

3. Compute the sensitivity and complimentary sensitivity function for a single plant in the example above ($k \in [1,5]$ plant). Can you prove why one has peaking in a lower frequency then the other. Try to generalize your answer for arbitrary stable systems (hint: plot generic margin bounds for S and T for same weight).