4. The Nichols Chart[1,2]

The Nichols chart

is a modified polar representation of complex numbers. Each complex number $s = x + iy \neq 0$ has a polar representation in terms of magnitude ρ and phase ϕ

The corresponding point on *NC* will be (ϕ, r) where $r = 20\log \rho$ in dB units.

1. James, H.M., Nichols, N.B., and Philips, R.S., *Theory of Servomechanisms*, 1st edn, McGraw-Hill. New York, 1947.
2. Cohen, N., et al., "Stability analysis using Nichols charts", *Int. J. Robust and Nonlinear Control*, Vol 4(3), pp. 3-20, 1994.

The map

transforms the Cartesian representation of *s* into its Nichols chart representation

4.1. M and N Circles[3]

Consider a unity feedback system written in terms of closed-loop magnitude and phase

$$T(j\omega) = \frac{L(j\omega)}{1+L(j\omega)} = Me^{j\phi}$$

and the open-loop in terms of real and imaginary parts

$$L(j\omega) = X + jY.$$

A constant-magnitude loci of T (M circle) is derived as follows

$$M = \left|\frac{X+jY}{1+X+jY}\right| \Rightarrow M^2 = \frac{X^2+Y^2}{(1+X)^2+Y^2}.$$

---

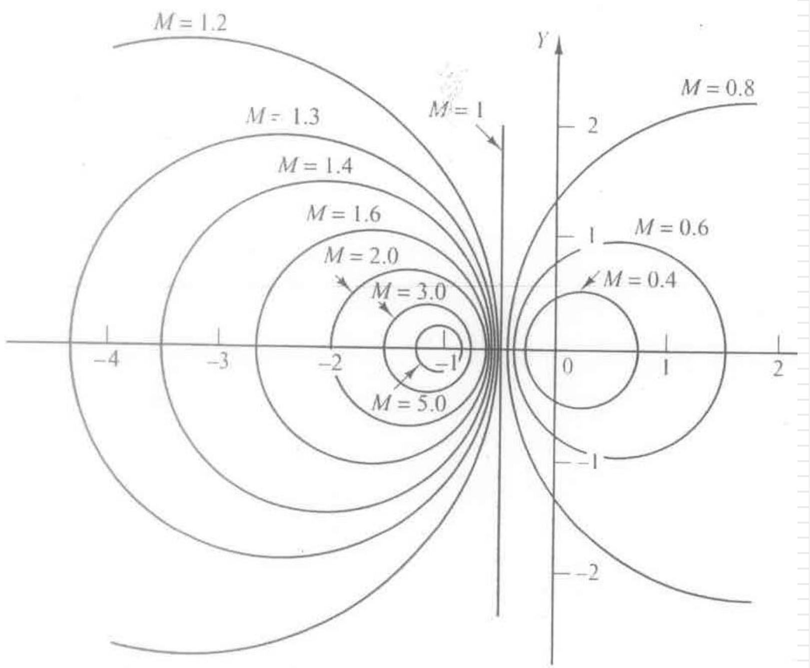
3. Ogata, K., *Modern Control Engineering*, 4th Ed., Prentice Hall, New jersey, 2002.

So

$$X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0.$$

There're 2 possibilities

$$\begin{cases} (X,Y)=\left(-\frac{1}{2},0\right) & M=1 \\ \left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2} & M\neq 1 \end{cases}$$

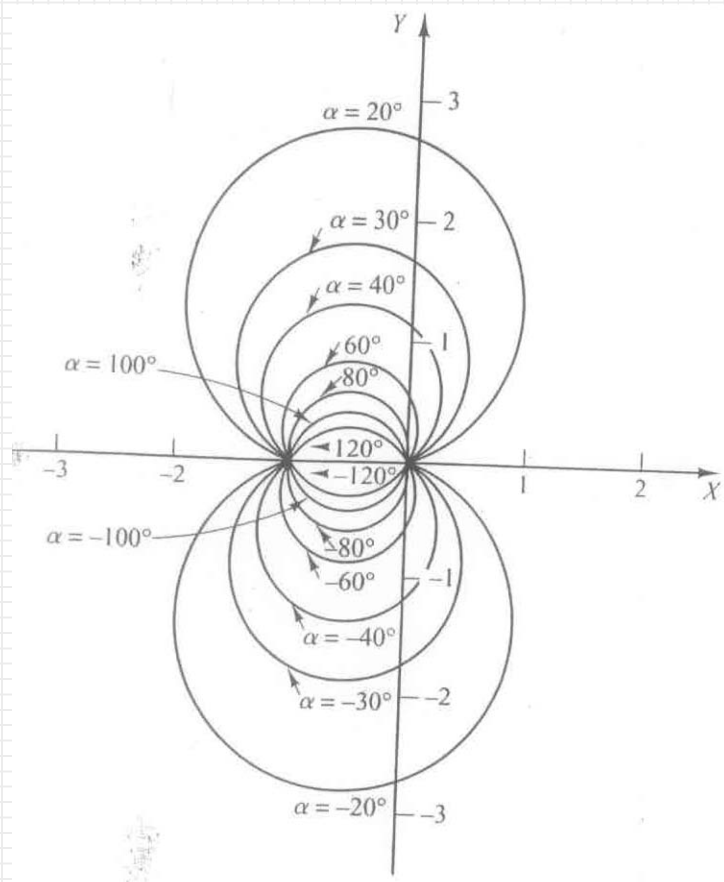


A constant-phase loci of T (N circle) is derived as follows

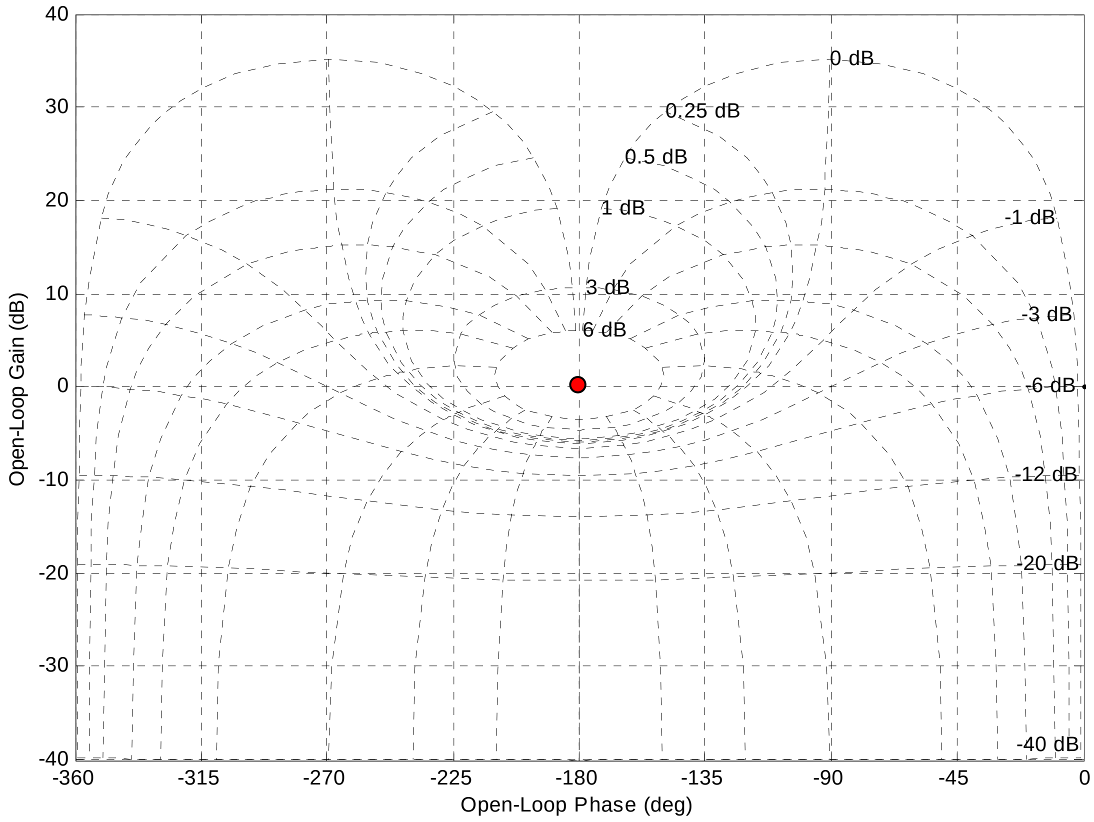
$$\angle T(j\omega) = \angle e^{j\phi} = \angle \frac{X + jY}{1 + X + jY} \Rightarrow \phi = \tan^{-1}\left(\tfrac{Y}{X}\right) - \tan^{-1}\left(\tfrac{Y}{1+X}\right).$$

Using $N = \tan\phi$, we obtain

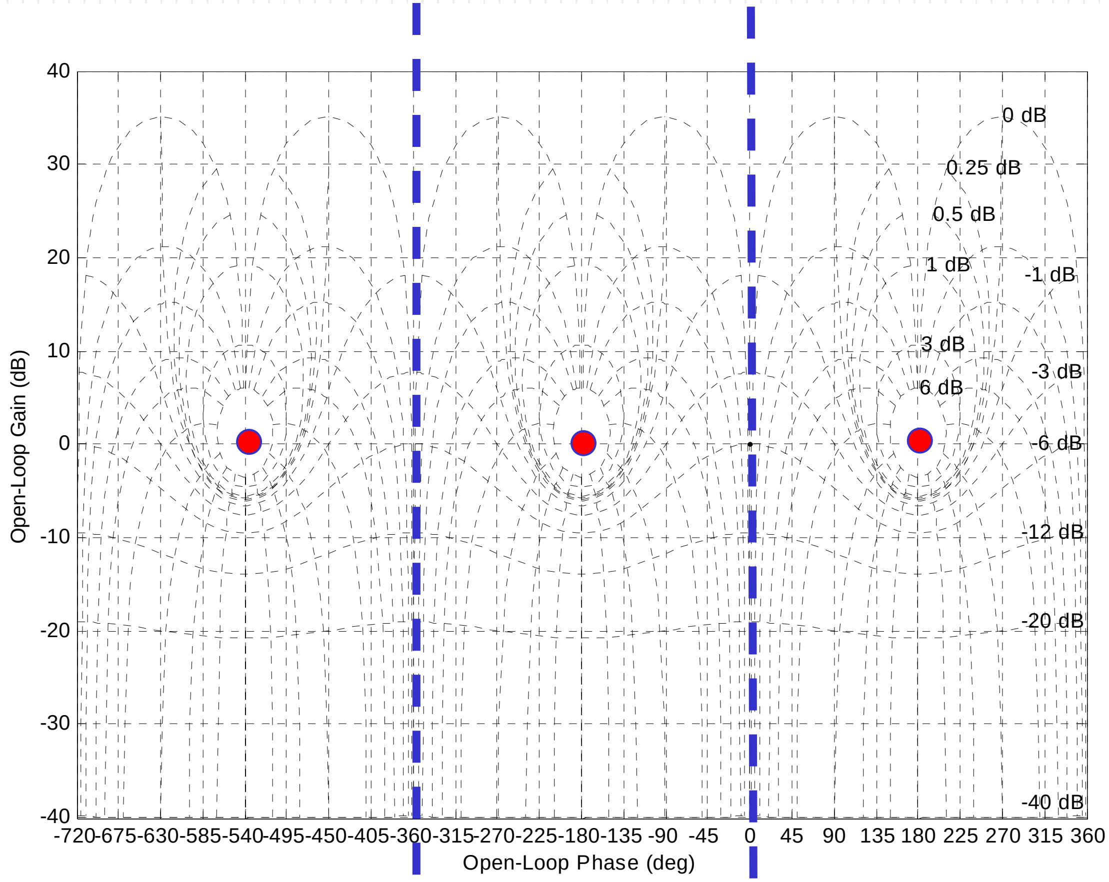
$$\left(X + \tfrac{1}{2}\right)^2 + \left(Y - \tfrac{1}{2N}\right)^2 = \tfrac{1}{4} + \left(\tfrac{1}{2N}\right)^2.$$



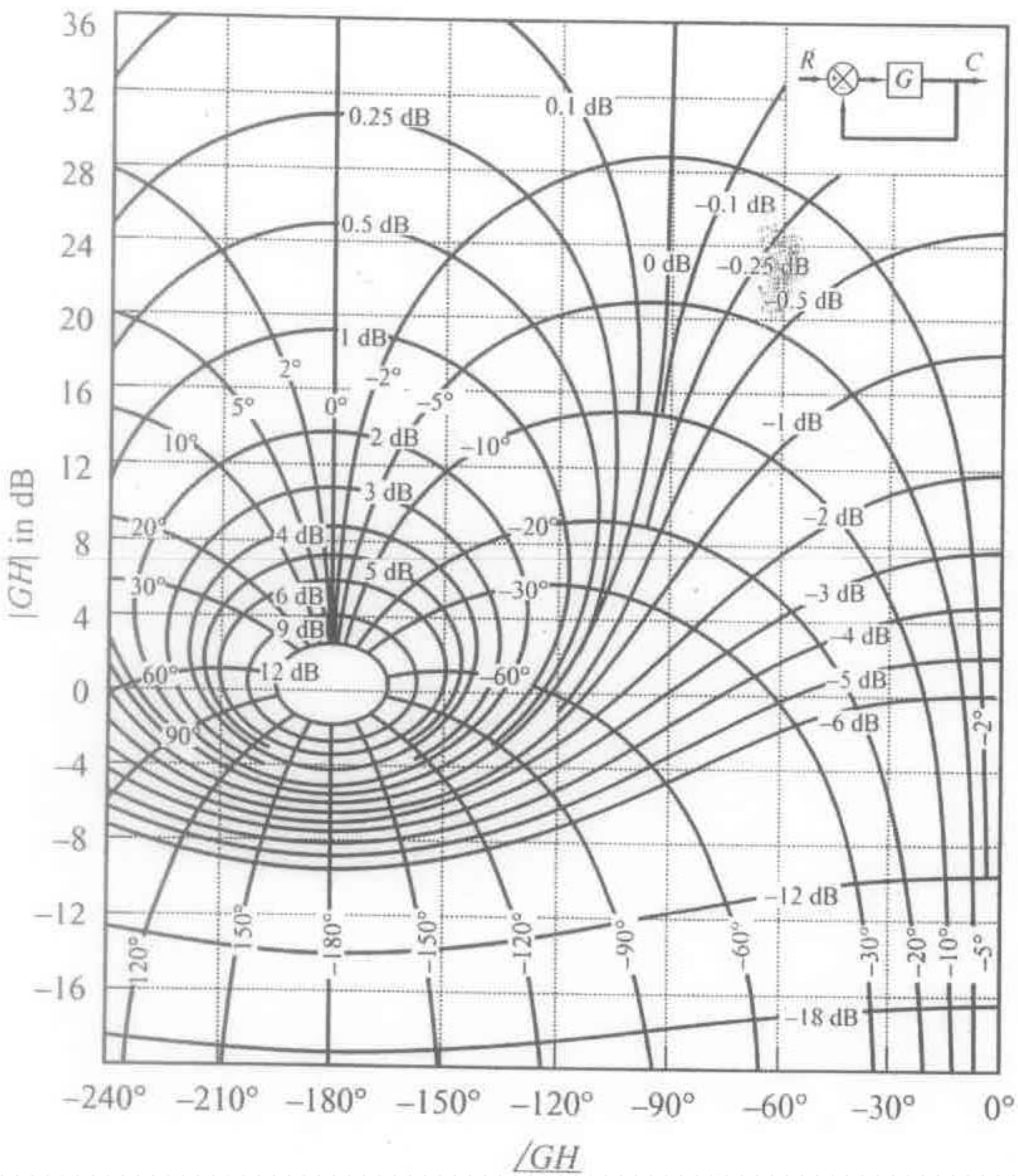
4.2. Examples of Nichols Chart

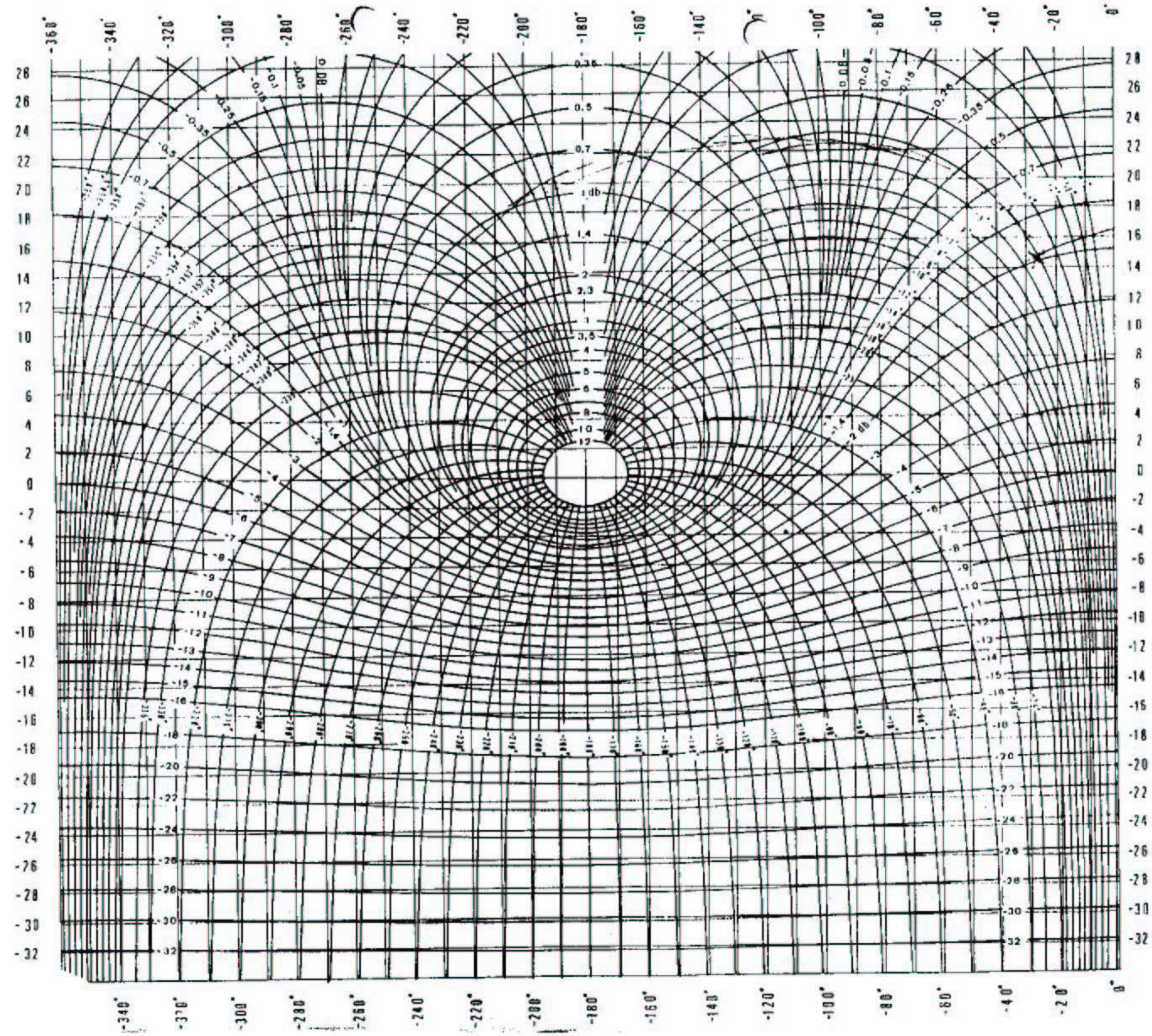


Single-sheeted Nichols chart

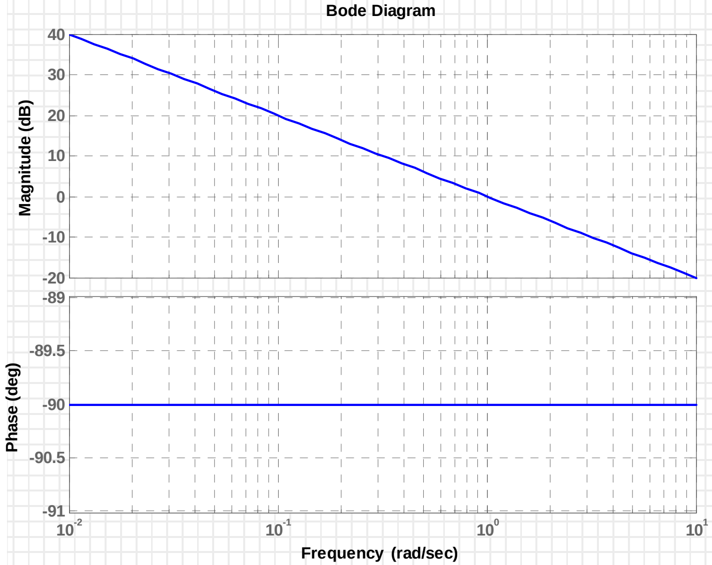


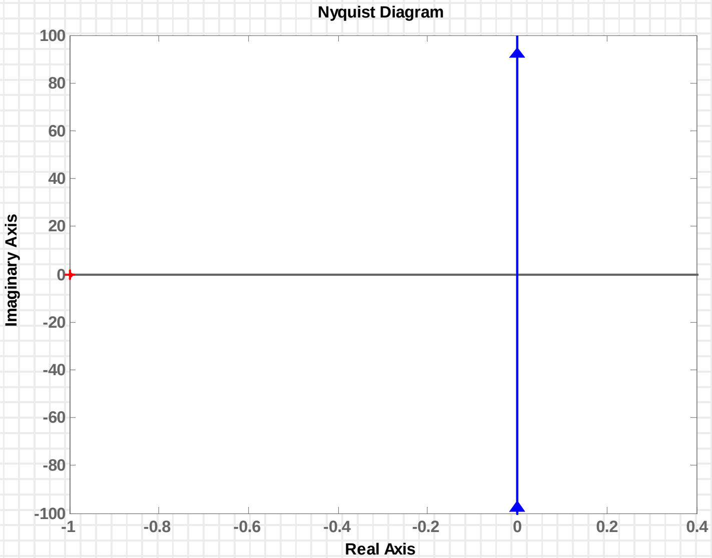
Multiple-sheeted Nichols chart

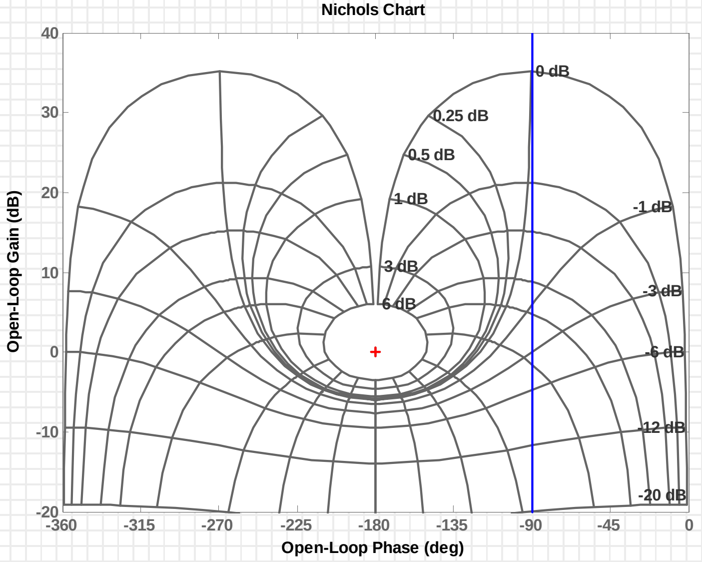

R
G
C
|GH| in dB
/GH
36
32
28
24
20
16
12
8
4
0
−4
−8
−12
−16
−240°
−210°
−180°
−150°
−120°
−90°
−60°
−30°
0°
0.25 dB
0.1 dB
0.5 dB
−0.1 dB
0 dB
−0.25 dB
−0.5 dB
1 dB
2 dB
3 dB
4 dB
5 dB
6 dB
9 dB
12 dB
−1 dB
−2 dB
−3 dB
−4 dB
−5 dB
−6 dB
−12 dB
−18 dB
2°
0°
−2°
5°
−5°
10°
−10°
20°
−20°
30°
−30°
60°
−60°
90°
−120°
−150°
−180°
−90°
−60°
−30°
−20°
−10°
−5°
−2°

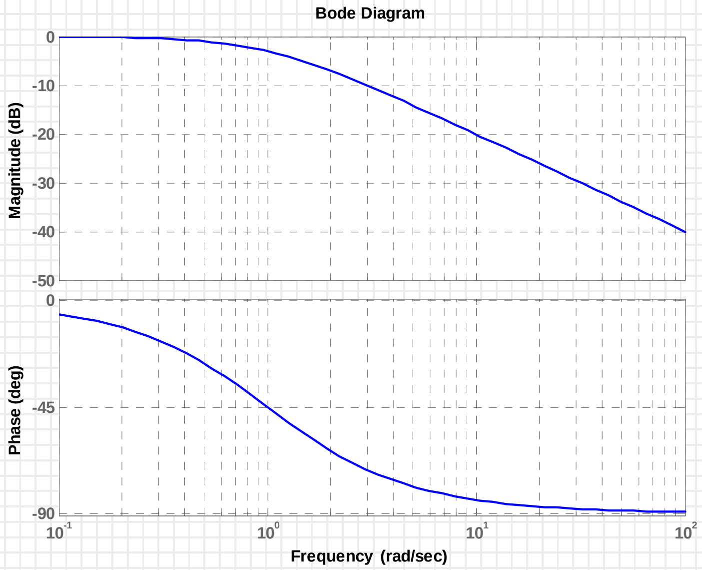

-360° -340° -320° -300° -280° -260° -240° -220° -200° -180° -160° -140° -120° -100° -80° -60° -40° -20° 0°
28 26 24 22 20 18 16 14 12 10 8 6 4 2 0 -2 -4 -6 -8 -10 -12 -14 -16 -18 -20 -22 -24 -26 -28 -30 -32
0.25 0.5 0.7 1 db 1.4 2 2.3 3 3.5 4 5 6 8 10 12
-1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15 -16 -18 -20 -22 -24 -26 -28 -30 -32

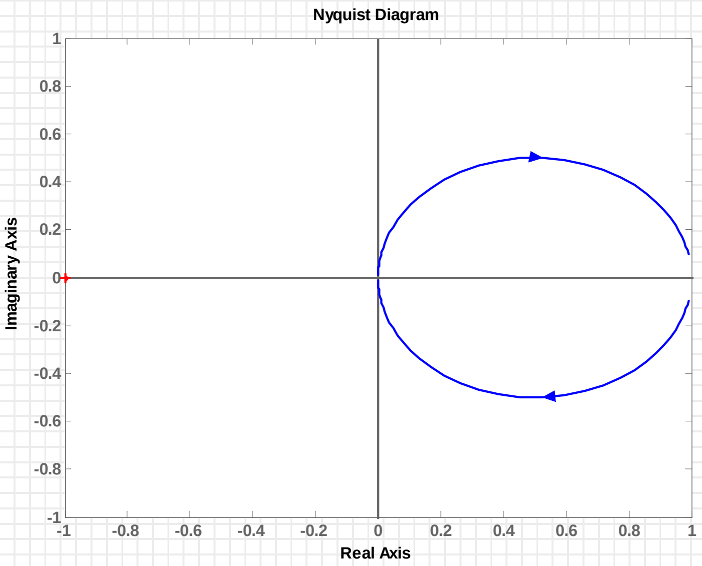
4.3. Frequency Response Plots

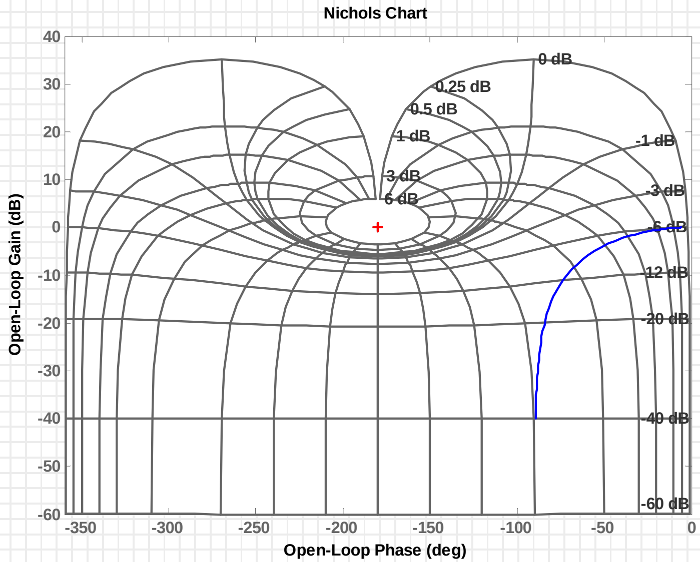


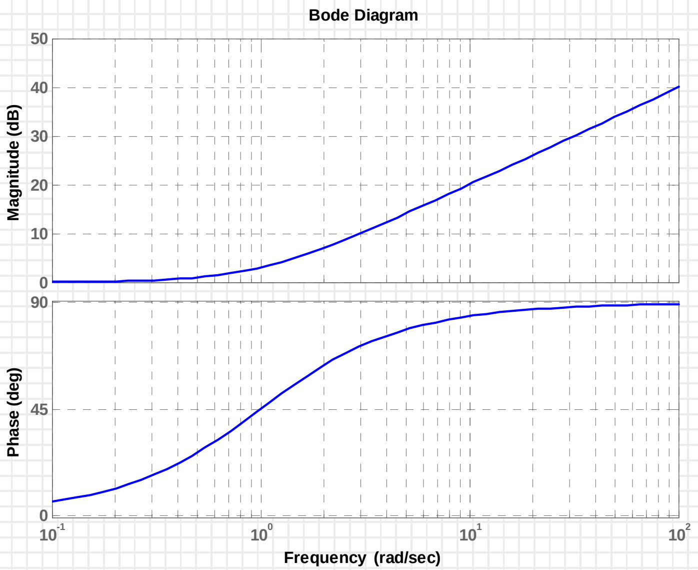


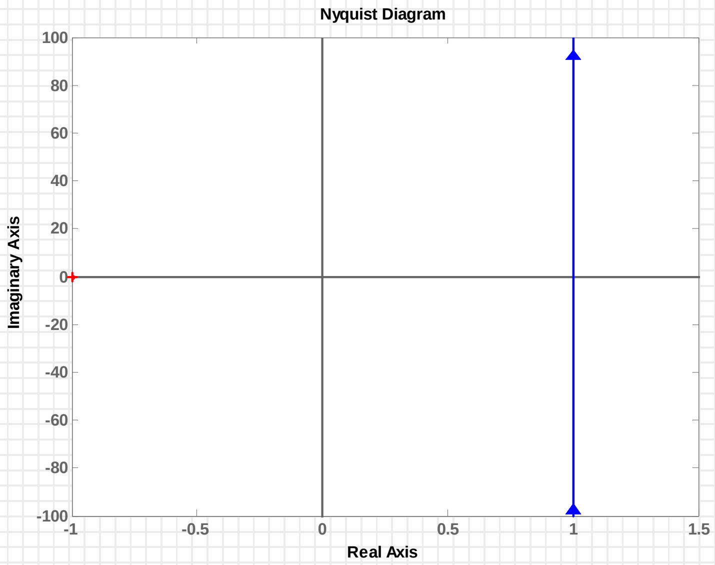


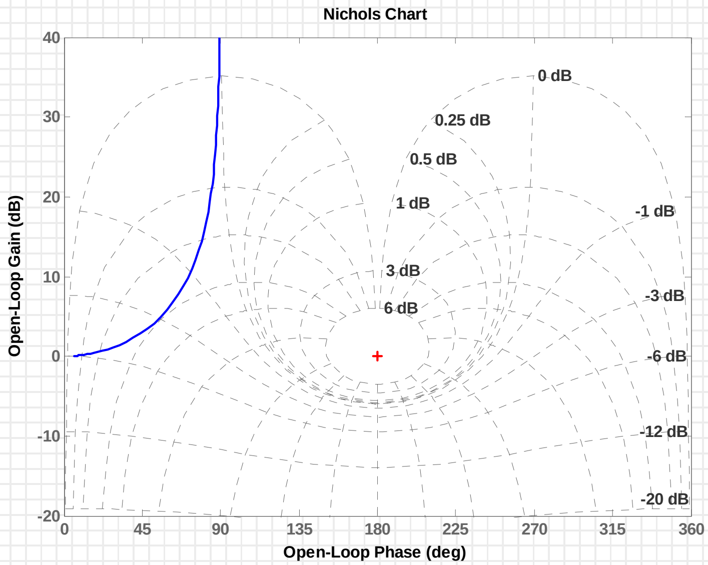

Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
0
-10
-20
-30
-40
-50
0
-45
-90
10^{-1}
10^{0}
10^{1}
10^{2}

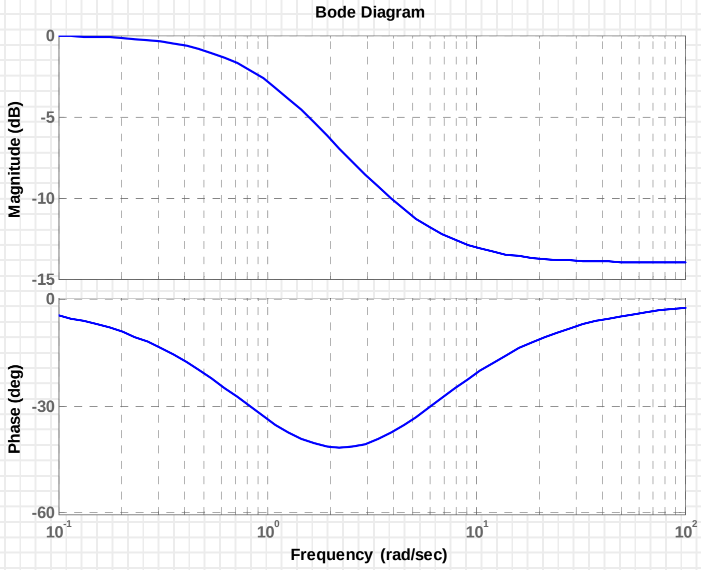

Nyquist Diagram
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB
-40 dB
-60 dB
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-350
-300
-250
-200
-150
-100
-50
0

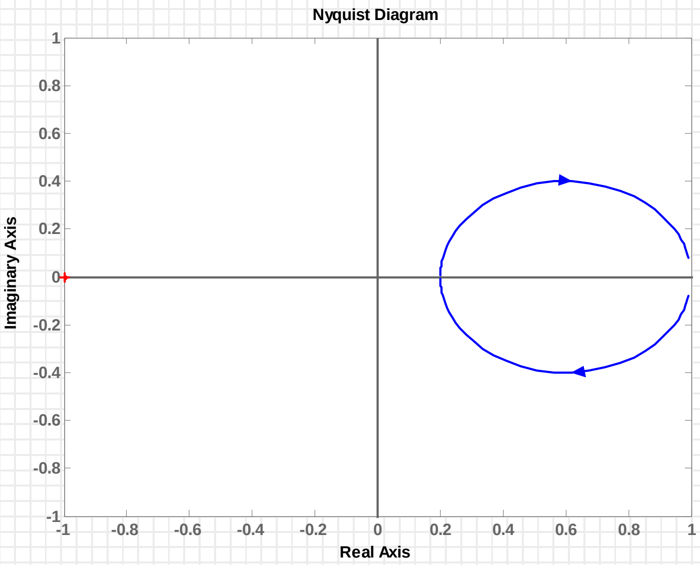

Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis

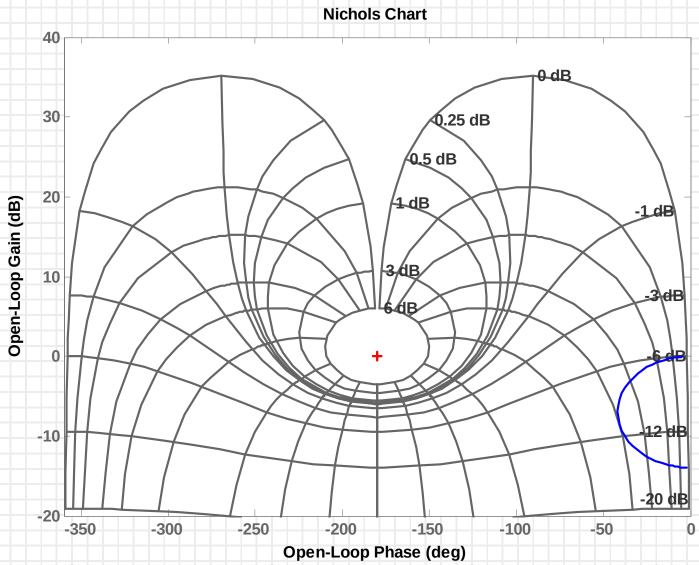

Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB



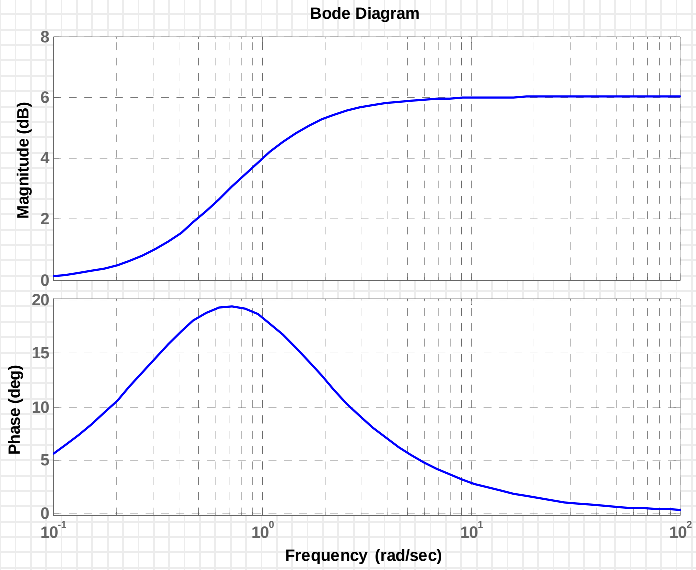
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)

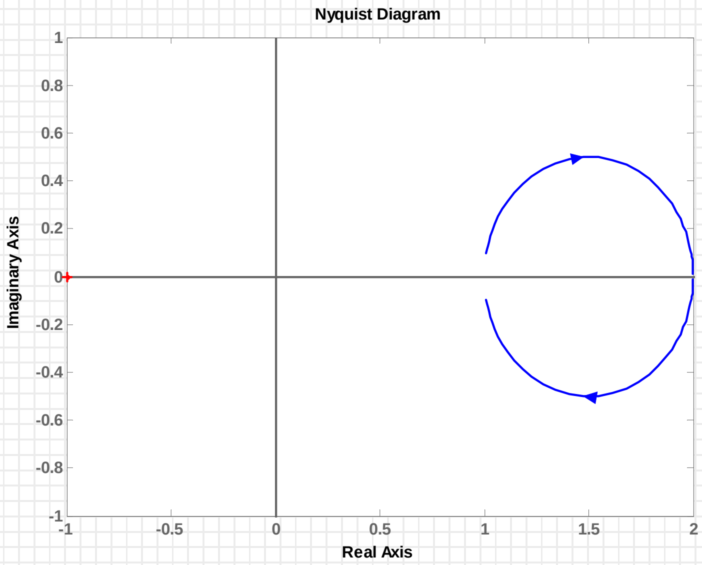


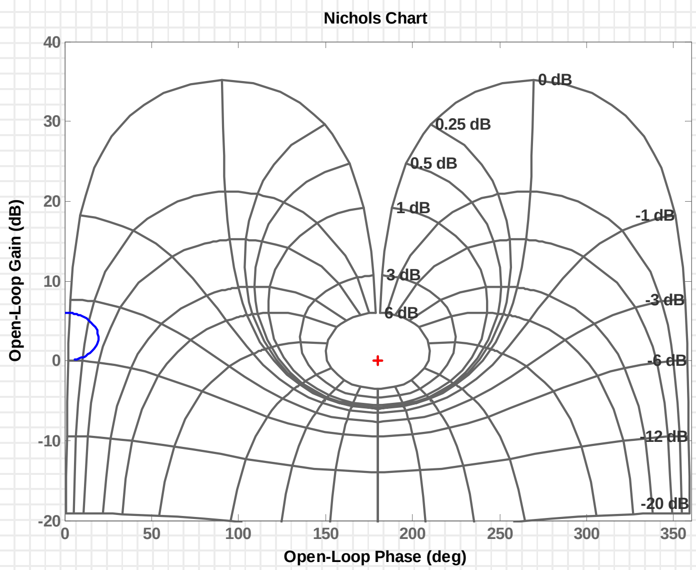
Nyquist Diagram
Imaginary Axis
Real Axis

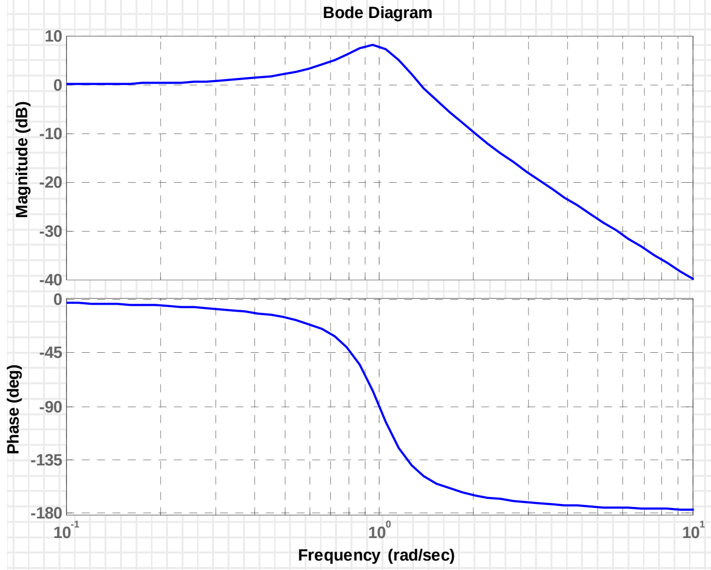


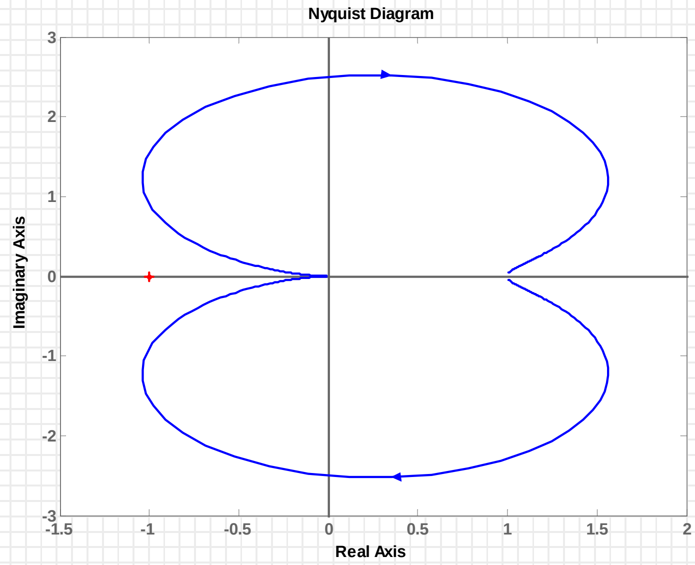
Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB

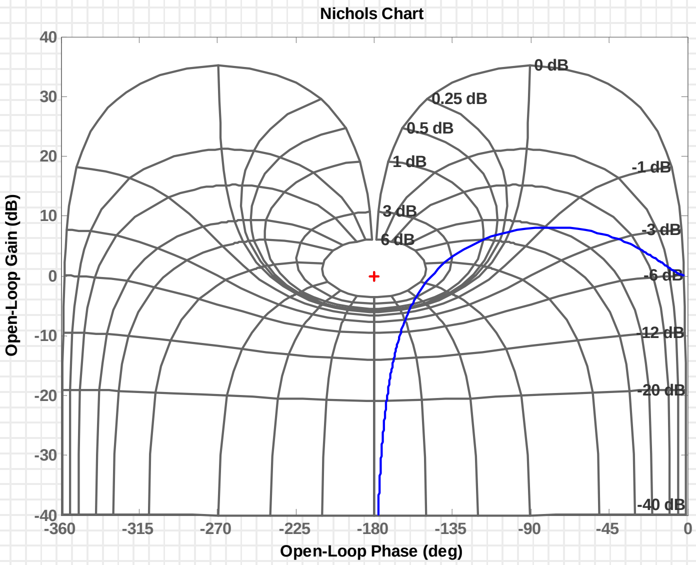

Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis

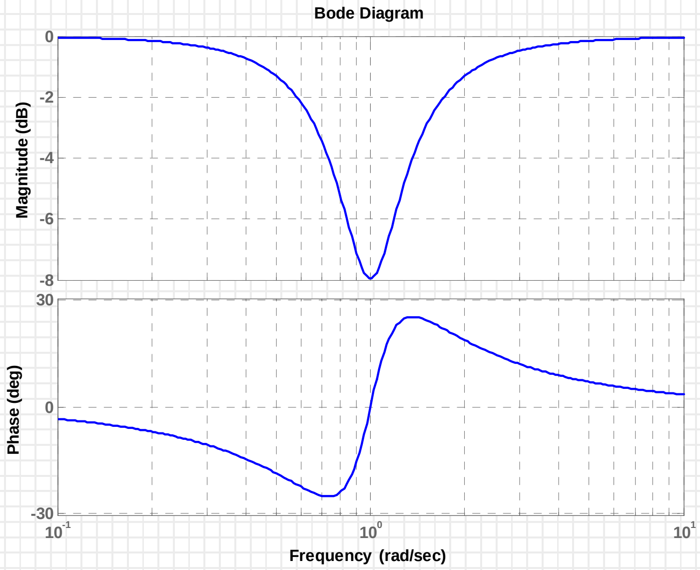

Nichols Chart
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB
Open-Loop Gain (dB)
Open-Loop Phase (deg)

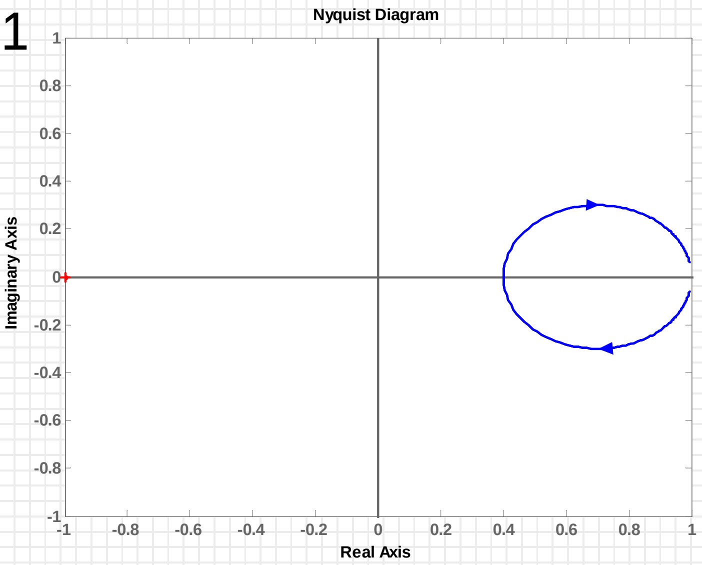

Bode Diagram
Magnitude (dB)
10
0
-10
-20
-30
-40
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

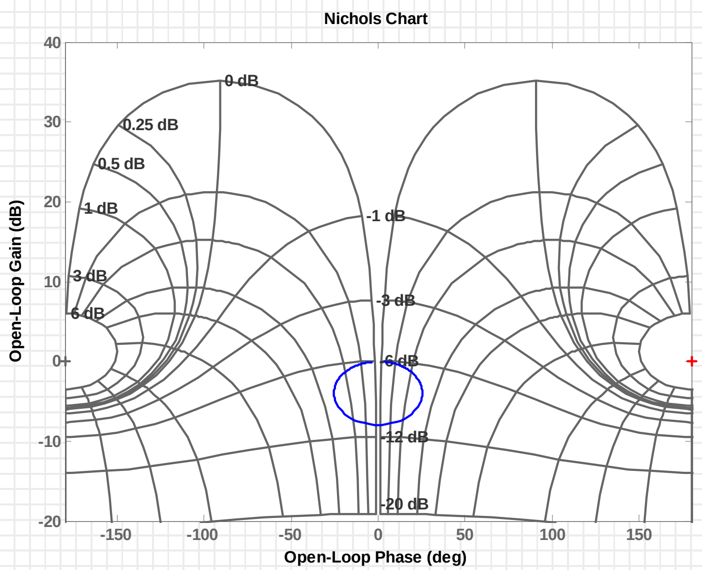

Nyquist Diagram
Imaginary Axis
Real Axis
3
2
1
0
-1
-2
-3
-1.5
-1
-0.5
0
0.5
1
1.5
2

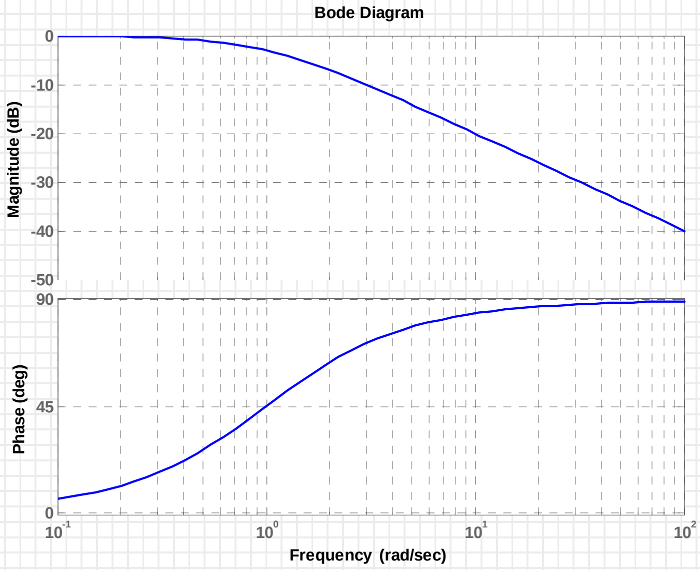

Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
40
30
20
10
0
-10
-20
-30
-40
-360
-315
-270
-225
-180
-135
-90
-45
0
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB
-40 dB

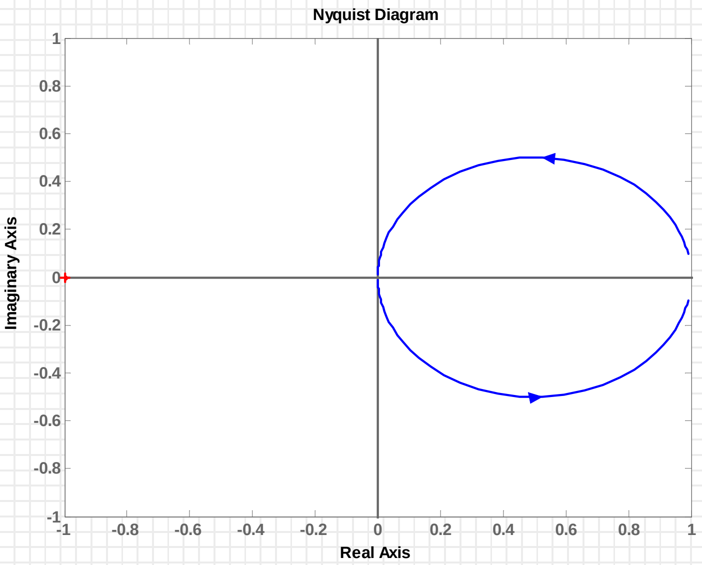
notch - $\dfrac{s^2 + 2\zeta_1\omega_n s + \omega_n^2}{s^2 + 2\zeta_2\omega_n s + \omega_n^2}, \zeta_2 < \zeta_1 < 1$

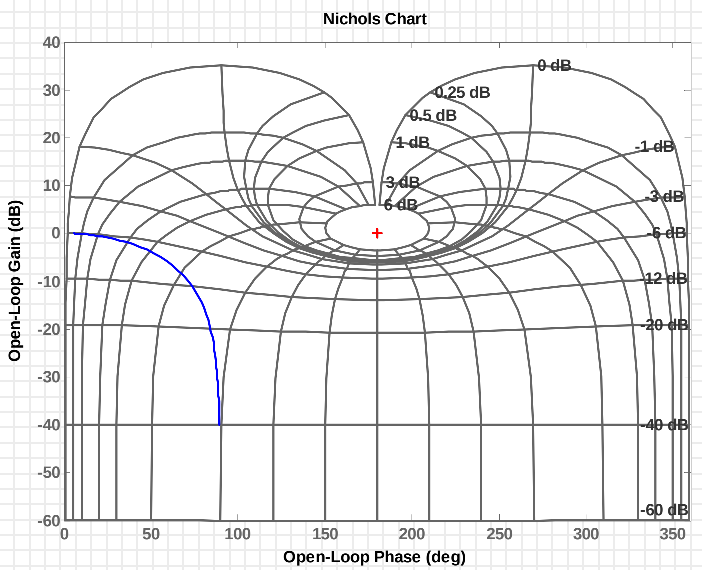


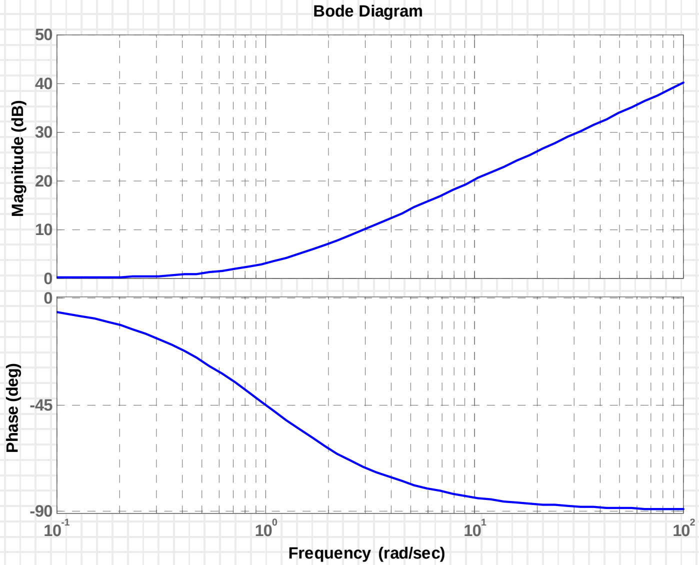


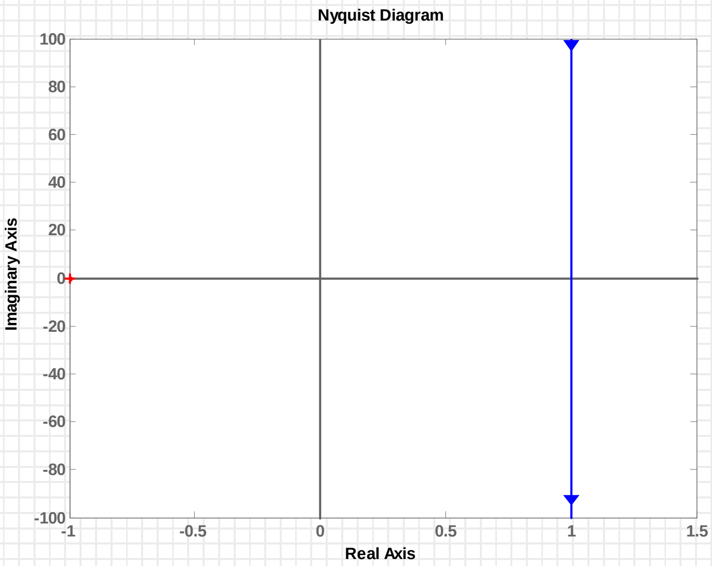


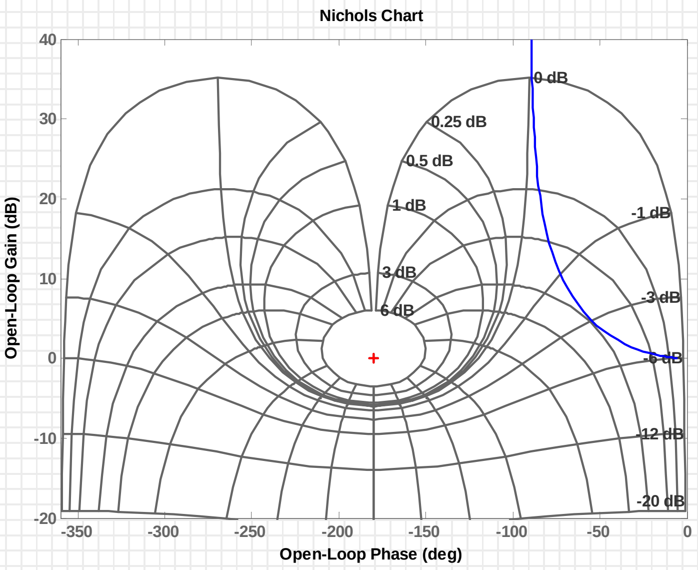

Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
Phase (deg)
90
45
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

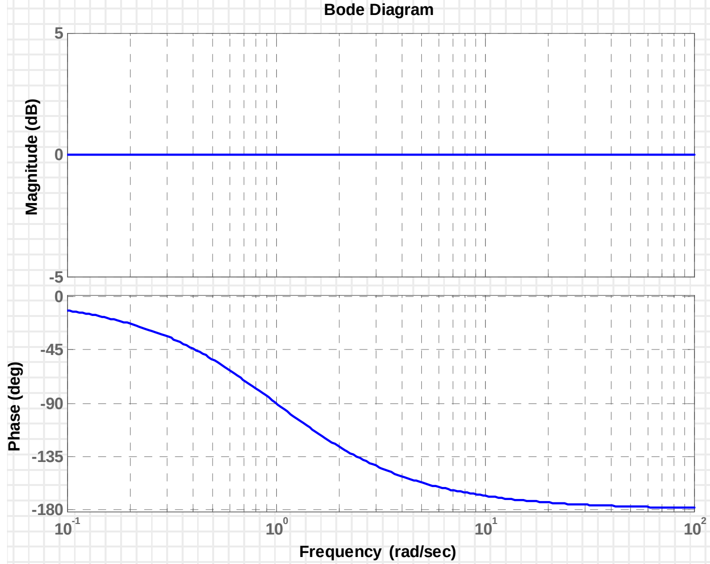

Nyquist Diagram
Imaginary Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Real Axis

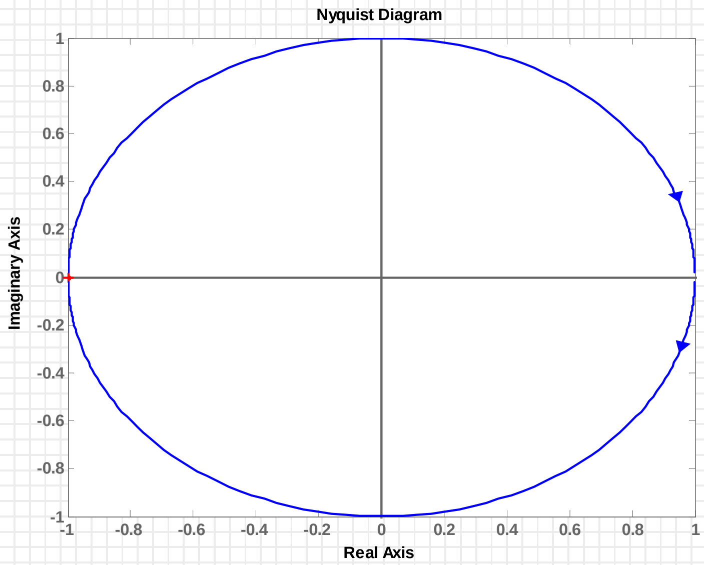

Nichols Chart
Open-Loop Gain (dB)
40
30
20
10
0
-10
-20
-30
-40
-50
-60
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB
-40 dB
-60 dB
0
50
100
150
200
250
300
350
Open-Loop Phase (deg)

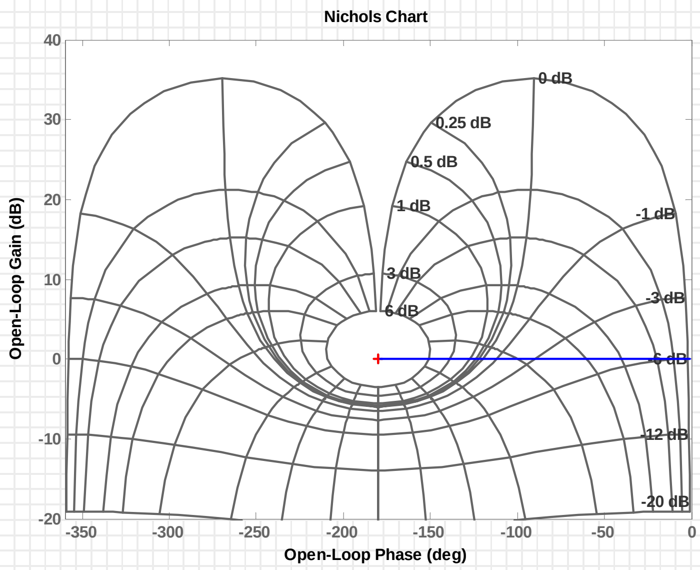

Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis


Nichols Chart
0 dB
0.25 dB
0.5 dB
1 dB
-1 dB
3 dB
-3 dB
6 dB
-6 dB
-12 dB
-20 dB
Open-Loop Gain (dB)
Open-Loop Phase (deg)


Bode Diagram
Magnitude (dB)
5
0
-5
Phase (deg)
0
-45
-90
-135
-180
10^-1
10^0
10^1
10^2
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Nichols Chart
Open-Loop Gain (dB)
Open-Loop Phase (deg)
40
30
20
10
0
-10
-20
-350
-300
-250
-200
-150
-100
-50
0
0 dB
0.25 dB
0.5 dB
1 dB
3 dB
6 dB
-1 dB
-3 dB
-6 dB
-12 dB
-20 dB

4.4. Exercises

Use MATLAB to plot the following frequency responses in Bode, Nyquist, and Nichols representations.

1. $$\frac{10(s+25)}{s(s+1)(s-100)(s^2+.1s+10)}$$

2. $$\frac{500e^{-.1s}(s+0.2)}{s(s^2+10s+100)}$$