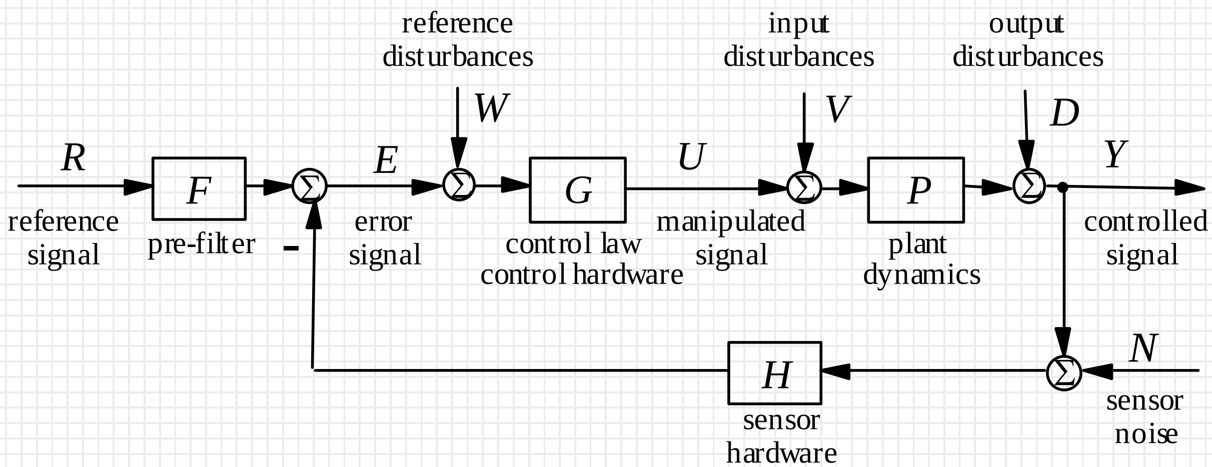
# 2. Generic SISO QFT Design

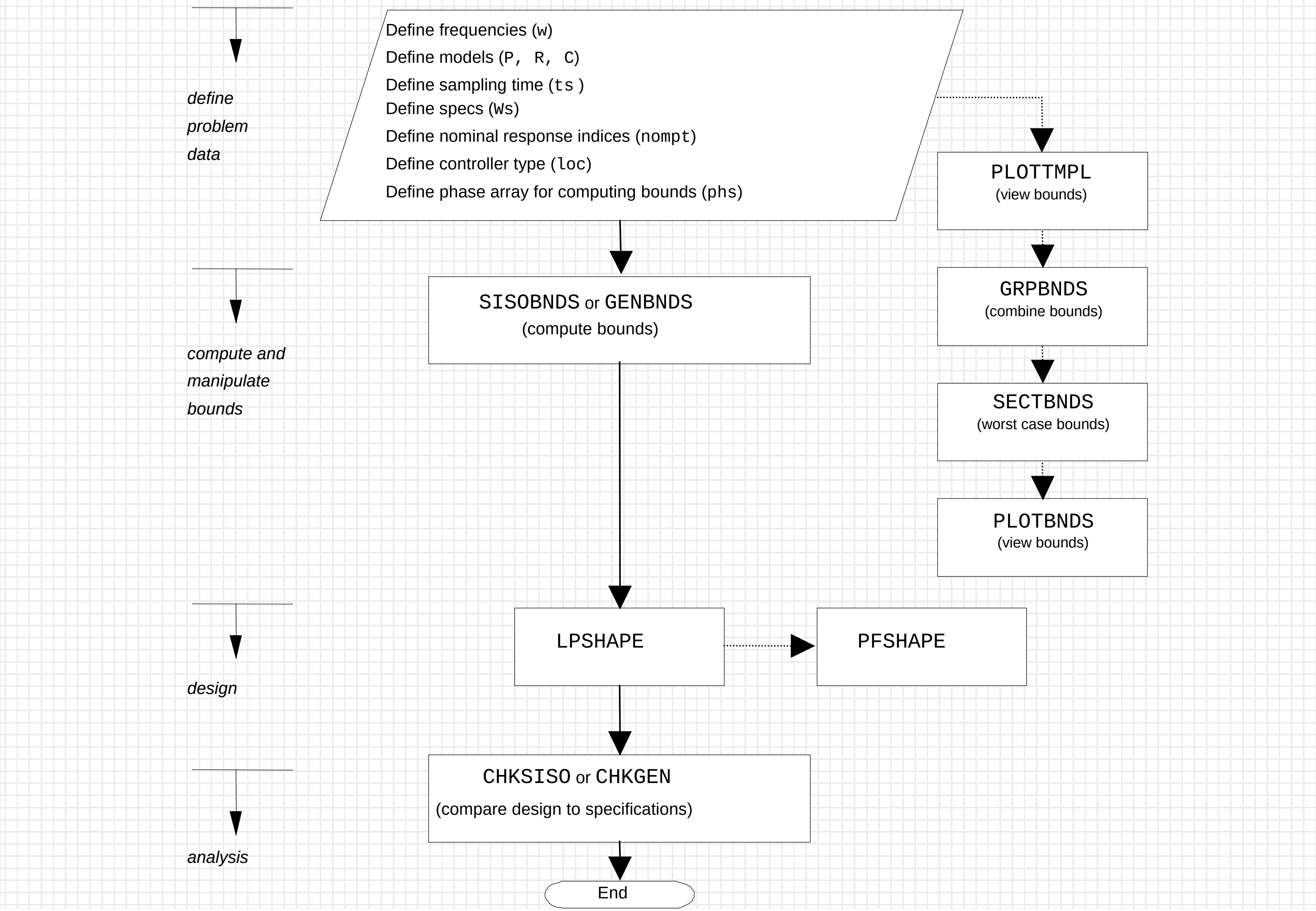
Consider the following feedback system (`qftex1.m`)




with an uncertain plant family $\mathcal{P}$

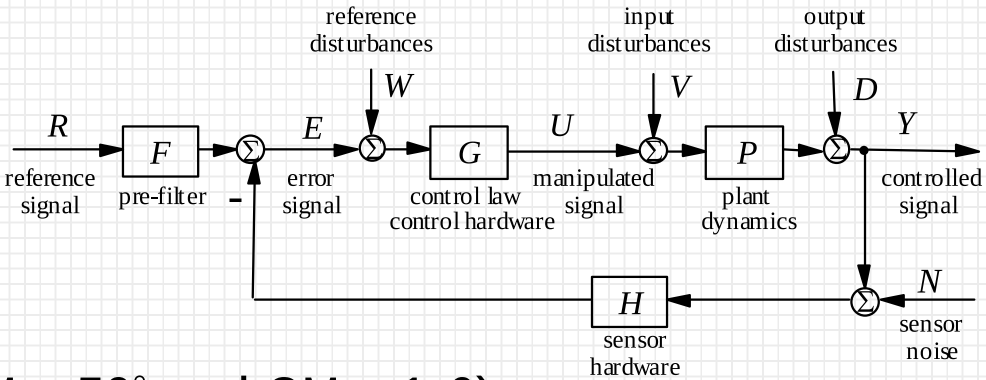
Assume $H$ = 1 and $F$ = 1.

# 2.0.1 QFT Design



# 2.1. Specifications

Performance specs.:



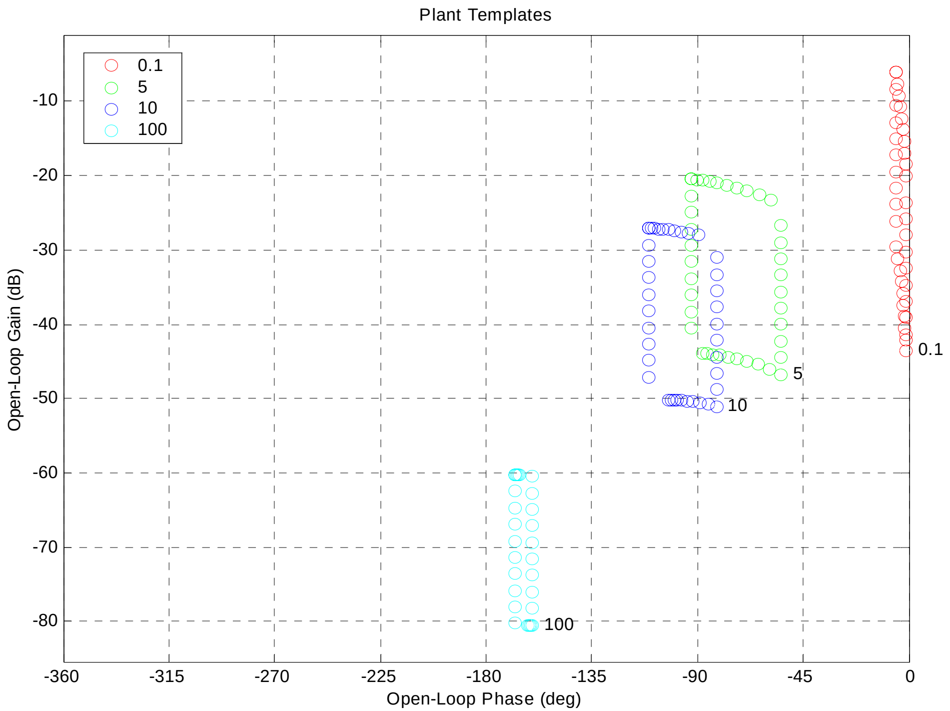

- Stability margins (PM = 50° and GM = 1.6)

- Output disturbance rejection

- Input disturbance rejection

# 2.2. Templates

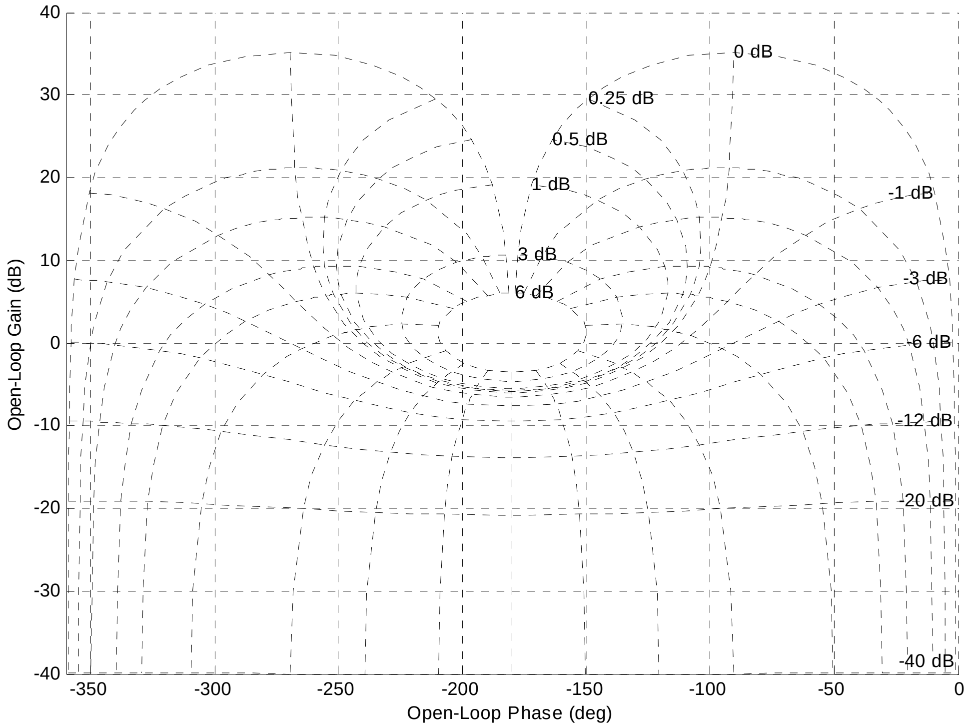
- Plot $P(j\omega)$: $\mathcal{P} = \left\{ P(s) = \frac{k}{(s+a)(s+b)} : \quad k \in [1,10], \ \ a \in [1,5], \ \ b \in [20,30] \right\}$



# 2.3. Bounds

Convert algebraic specs into equivalent specs in terms on <u>nominal loop</u> $L_0=GP_0H_0$. For example, consider the classical $M$ and $N$ circles plotted on a Nichols chart.

$$\left|T_0(j\omega)\right| = \left|\frac{L_0(j\omega)}{1+L_0(j\omega)}\right| \le M$$

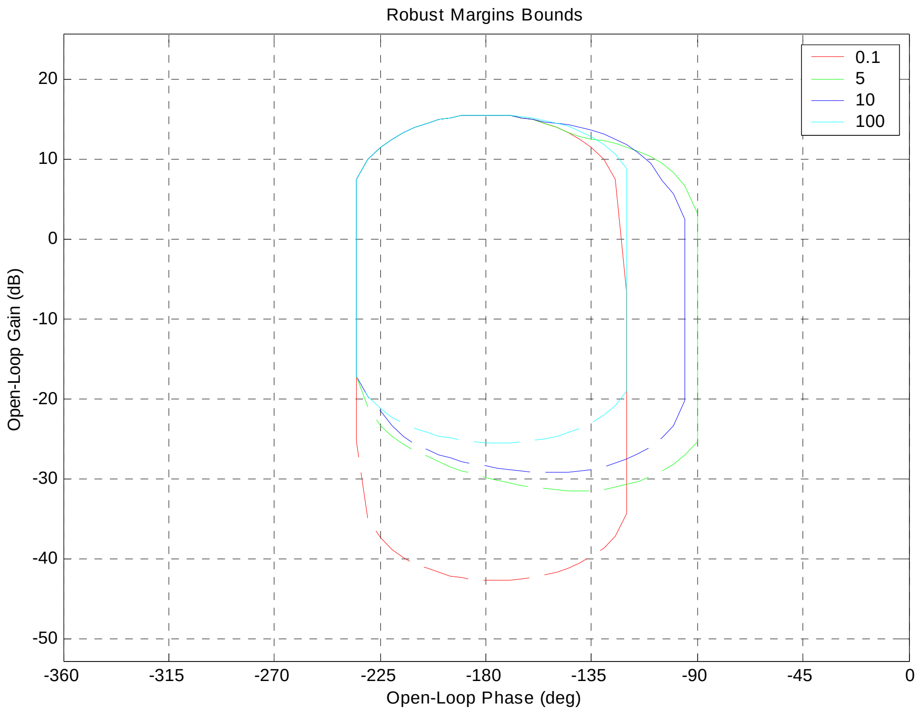


# 2.3.1. Margin Bounds

Convert robust stability margin spec in terms of nominal loop.

$$L_0 = GP_0,\, P_0 \in \mathcal{P}$$

$$|T(j\omega)| = \left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq 1.2, \quad \forall P \in \mathcal{P}$$

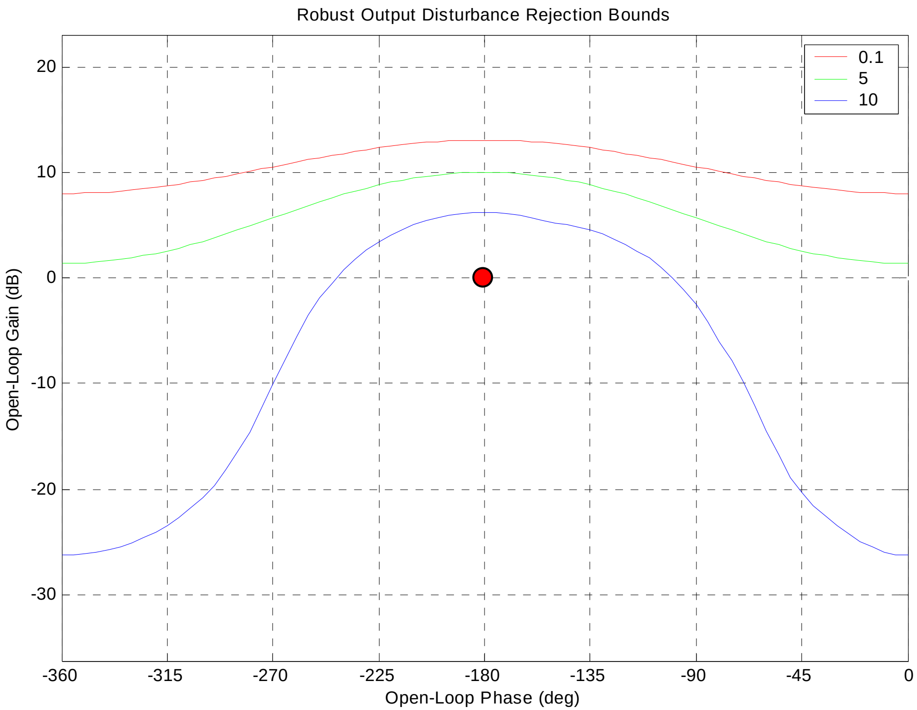


# 2.3.2. Output Disturbance Bounds

Convert robust output disturbance spec in terms of nominal loop.

$$\left|\frac{Y(j\omega)}{D(j\omega)}\right| \leq W(\omega), \text{ for all } P \in \mathcal{P}, \ \omega \in [0,10)$$

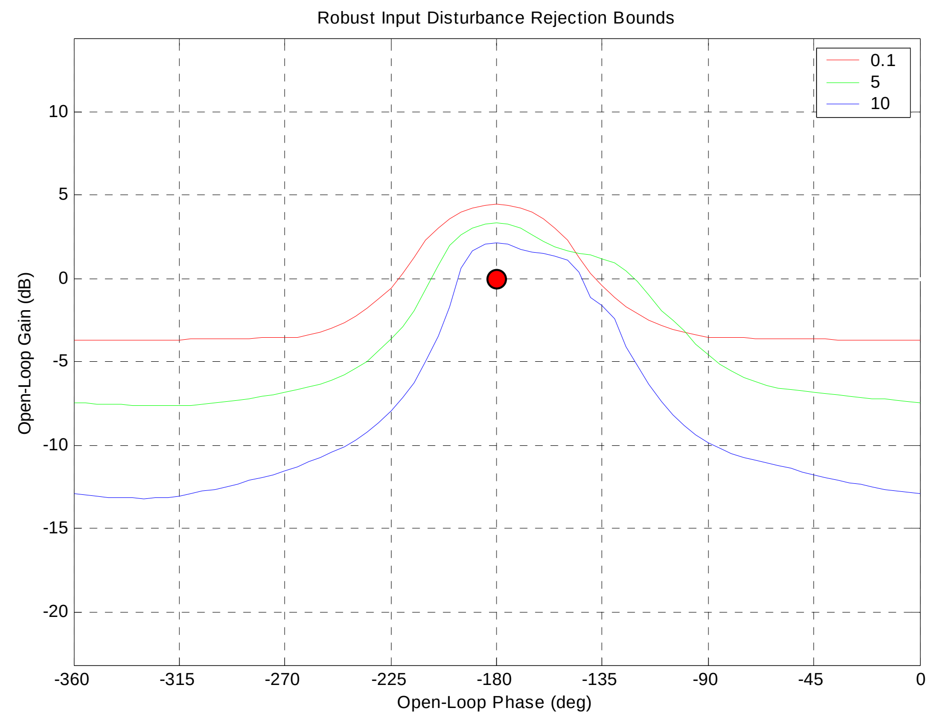


# 2.3.3. Input Disturbance Bounds

Convert robust input disturbance spec in terms of nominal loop.

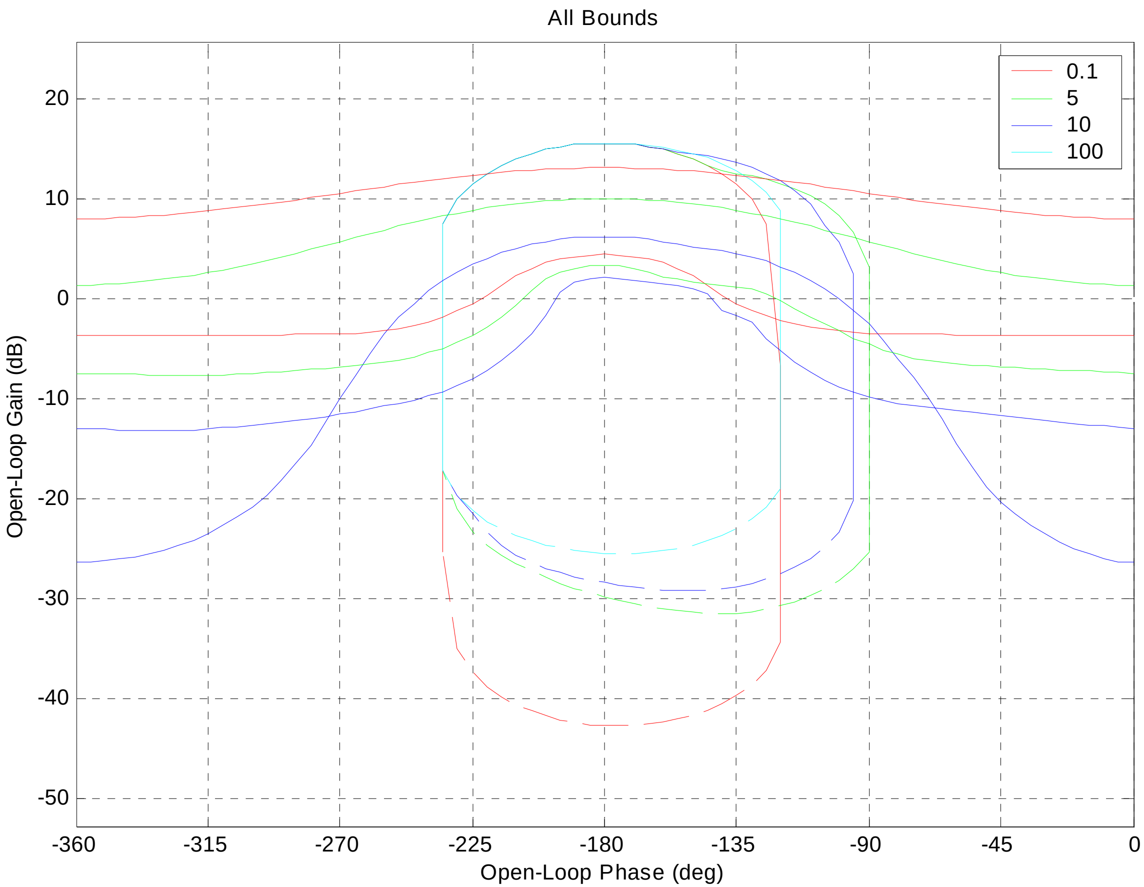
$$\left|\frac{Y(j\omega)}{V(j\omega)}\right| \le 0.01, \text{ for all } P \in \mathcal{P}, \ \omega \in [0,50)$$



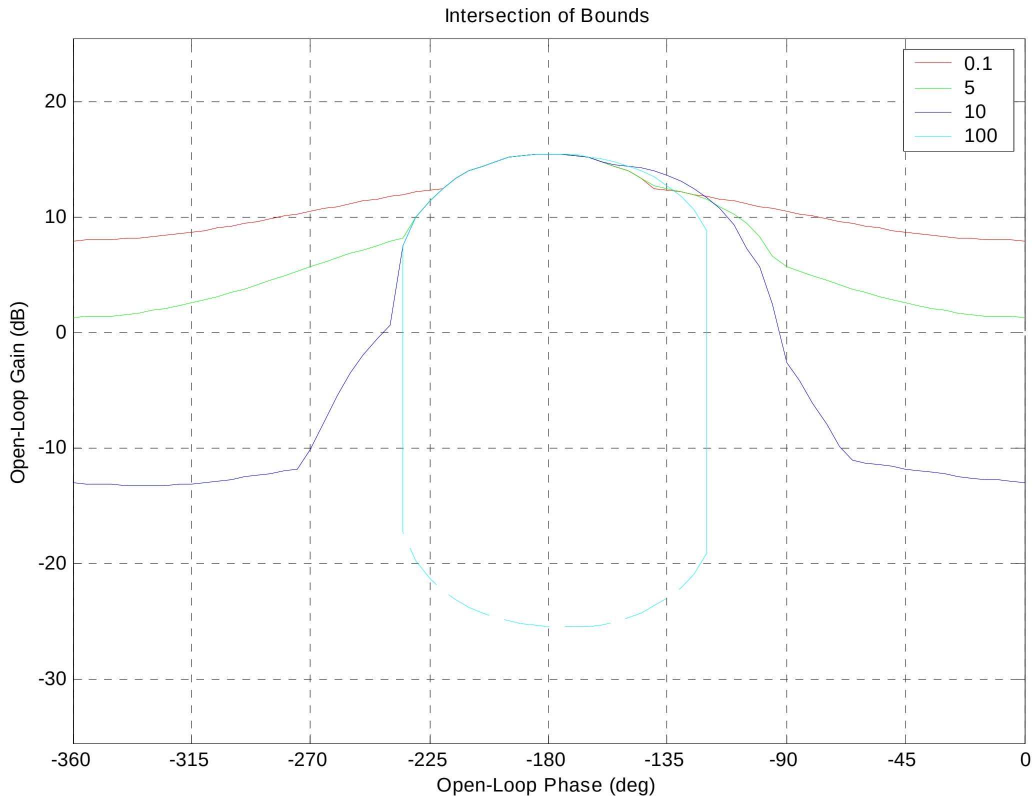
# 2.3.4. All Bounds

View all bounds.



# 2.3.5. Effective Bounds

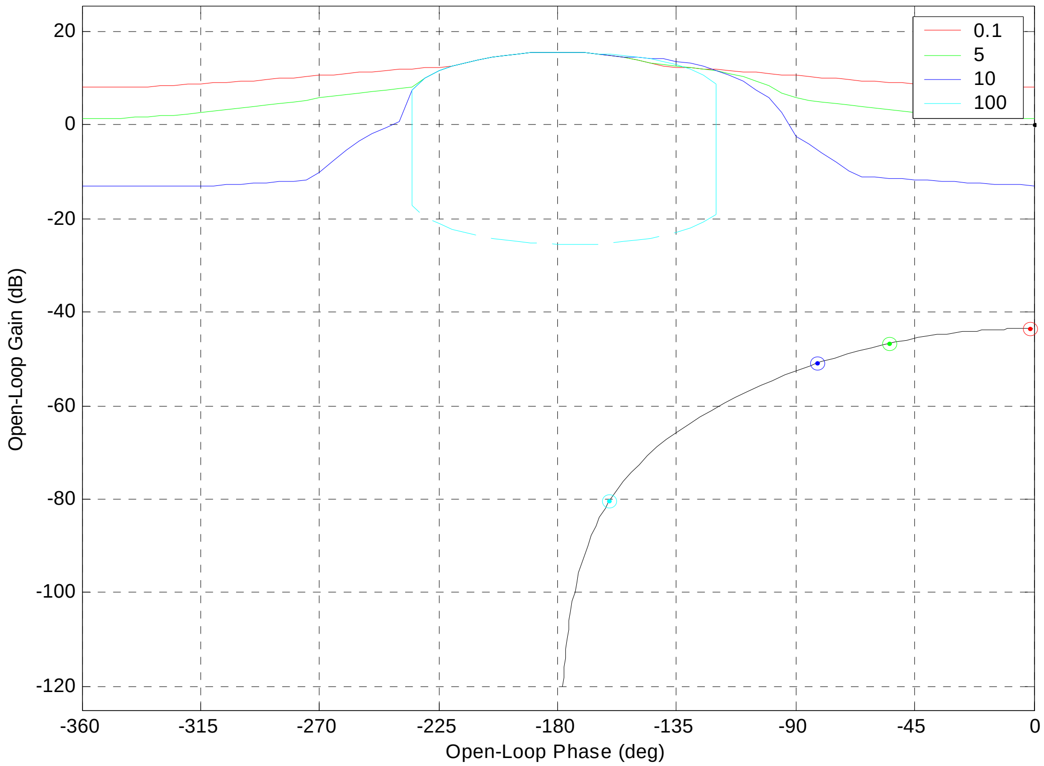
Compute worst-case bound by computing their



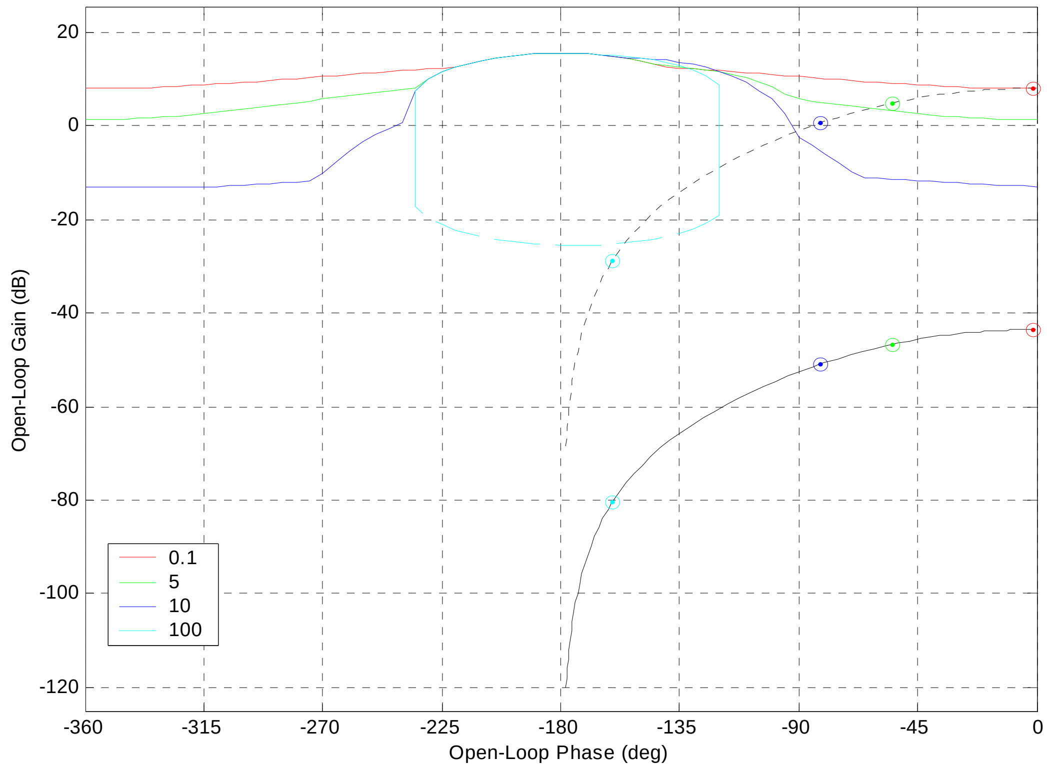
# 2.4. Loop Shaping

Plot nominal loop $L_0 = GP_0,\ P_0 \in \mathcal{P}$.

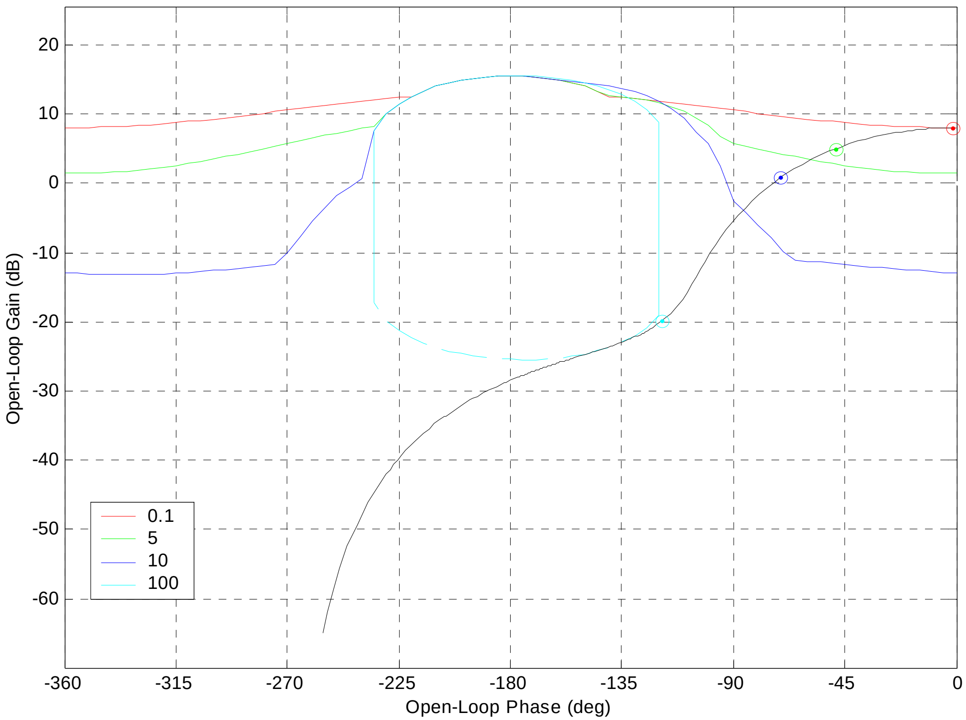
Start with $G = 1$



# 2.4.1. Loop Shaping: step 1



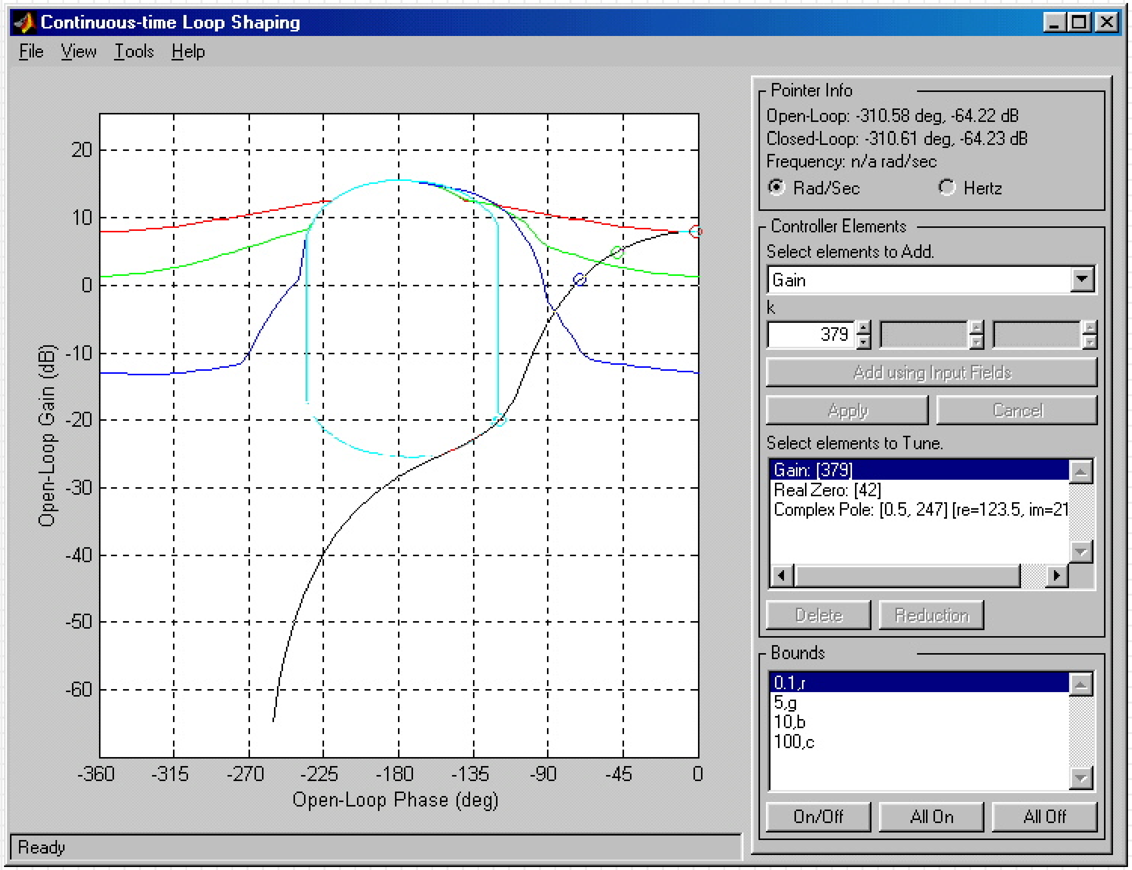
# 2.4.2. Loop Shaping: final steps



# 2.4.3. Loop Shaping Environment

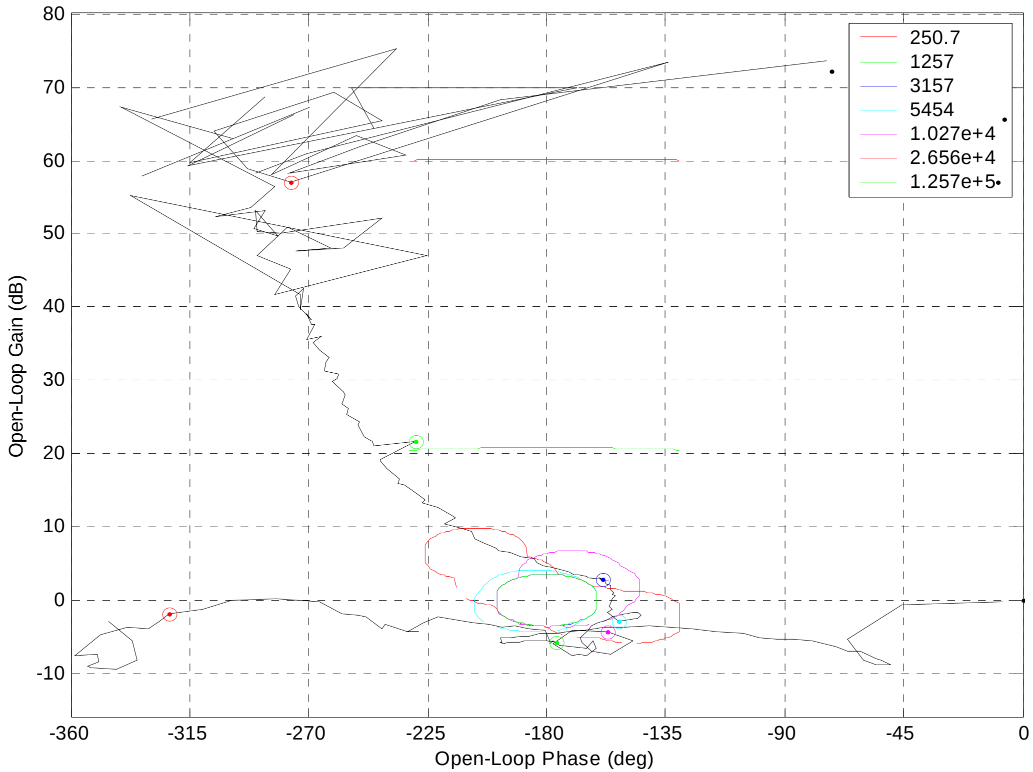
`lpshape`
screen capture




$$L_0 = 379\,\frac{\frac{s}{42}+1}{\frac{s^2}{250^2}+\frac{2\cdot 0.5\cdot s}{250}+1}\cdot\frac{1}{(s+1)(s+10)}$$

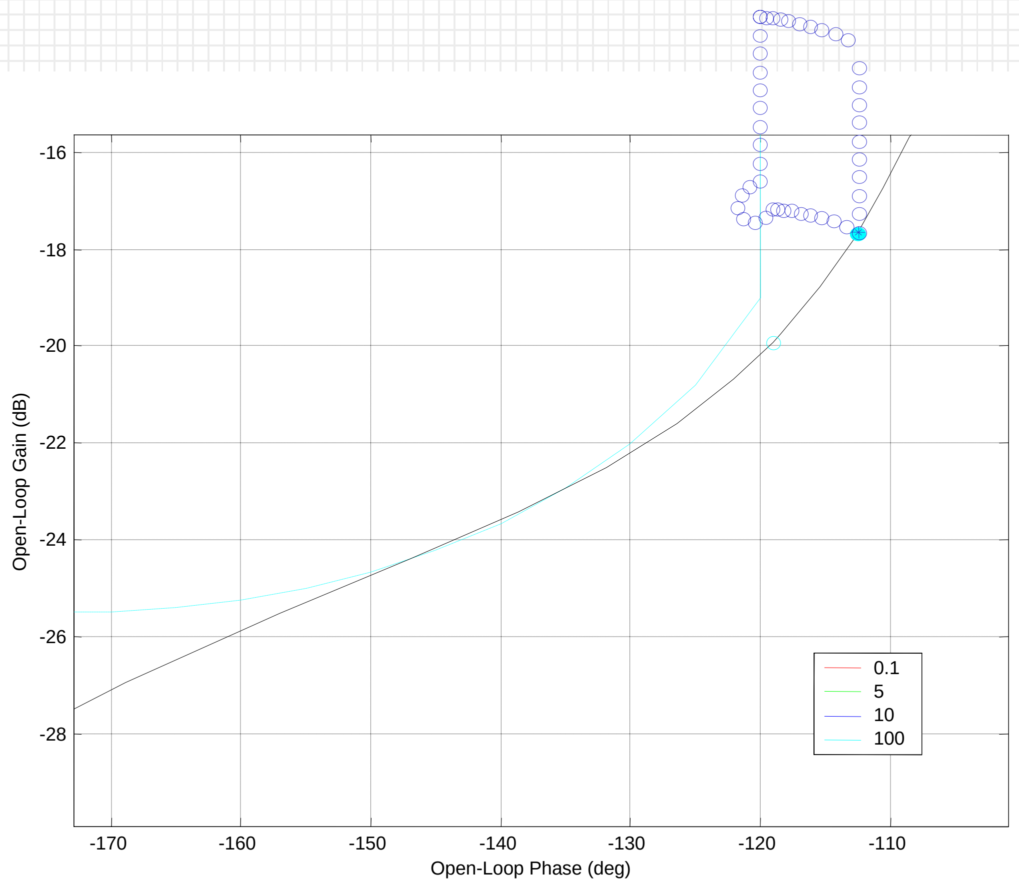
# 2.4.3. Working with Measured Data




Radial loop in a CD ROM drive:
showing measured freq response.

# 2.5. Analysis

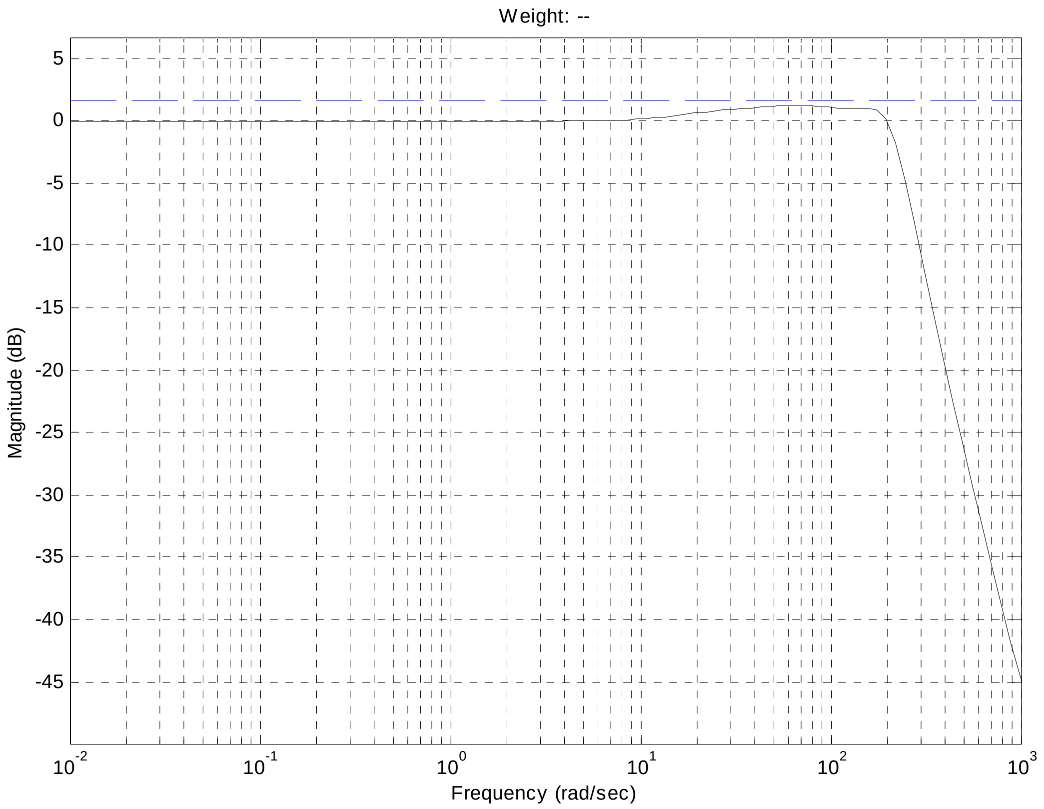
Verify closed-loop performance with the designed controller. Why?



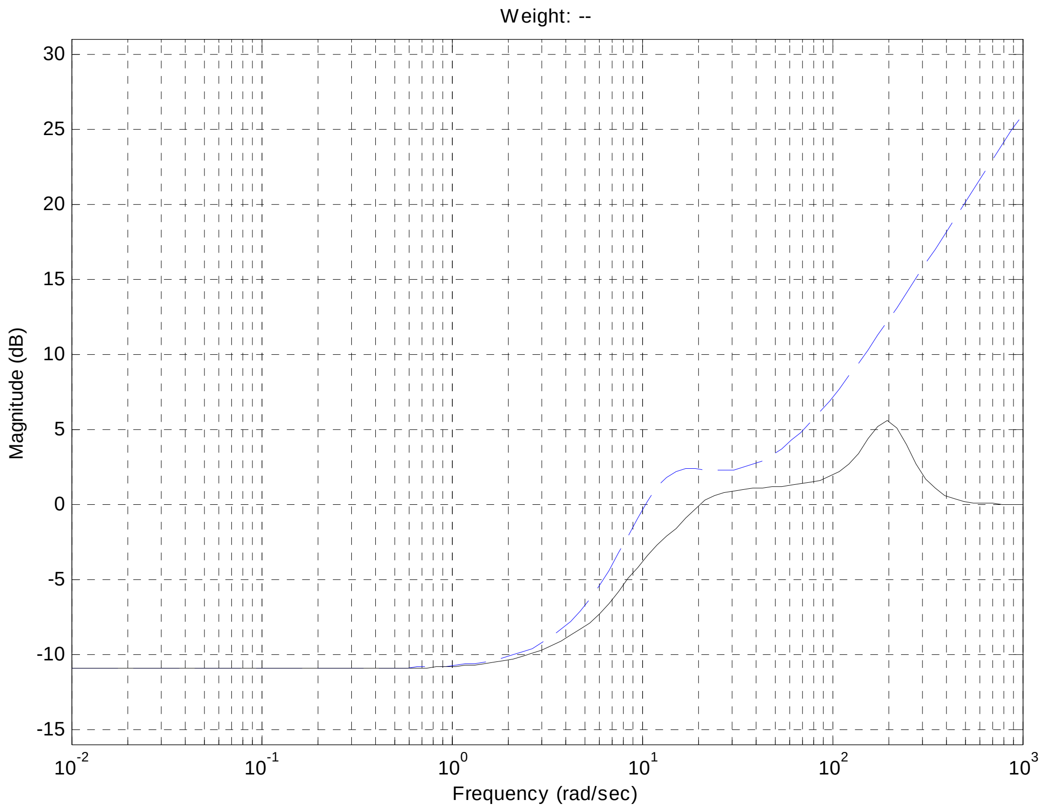
# 2.5.1. Analysis: Margins

$$\max_{P \in \mathcal{P}} \left| \frac{PG(j\omega)}{1+PG(j\omega)} \right| \qquad 1.58dB\,(1.2)$$



# 2.5.2. Analysis: Output Disturbance

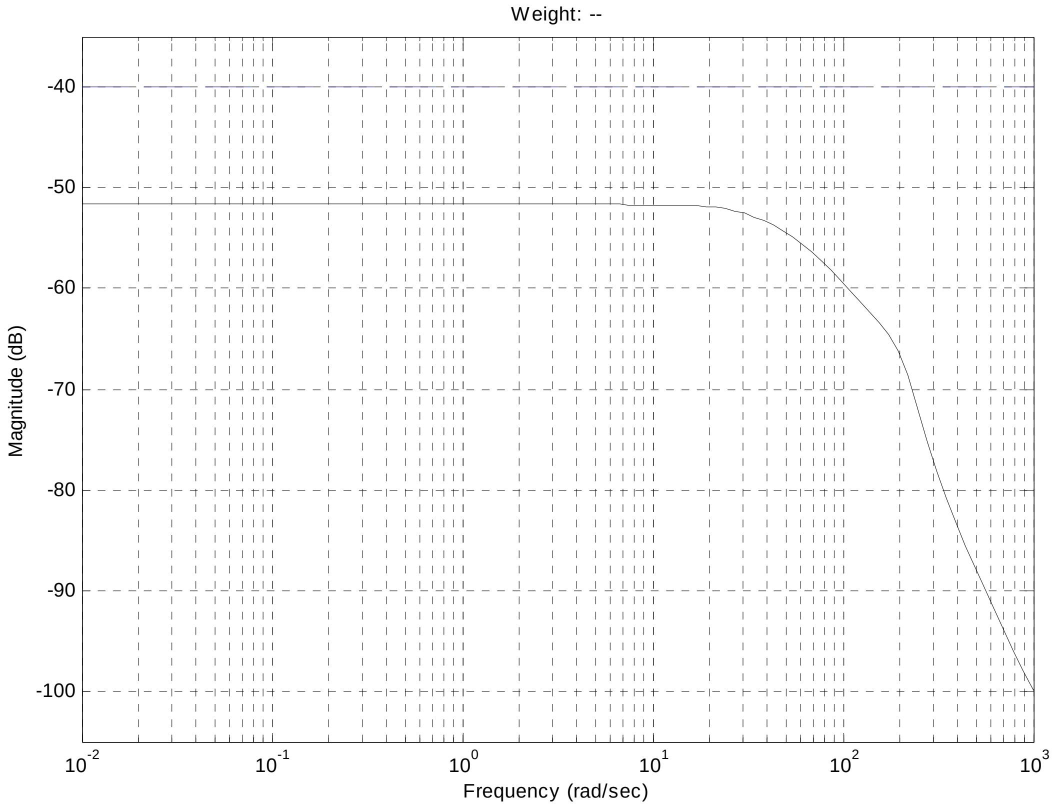
$$\left| 0.02 \frac{(j\omega)^3 + 64(j\omega)^2 + 748(j\omega) + 2400}{(j\omega)^2 + 14.4(j\omega) + 169} \right|$$




$$\max_{P \in \mathcal{P}} \left| \frac{1}{1 + PG(j\omega)} \right|$$

# 2.5.3. Analysis: Input Disturbance

$$-40dB\,(0.01)$$




$$\max_{P\in\mathcal{P}}\left|\frac{1}{1+PG(j\omega)}\right|$$