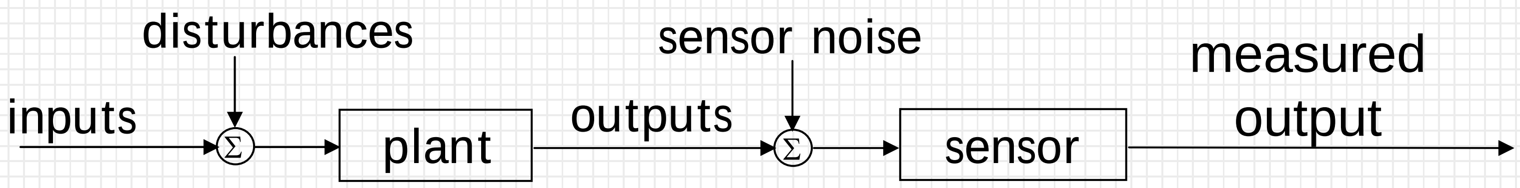
1. Why Feedback?

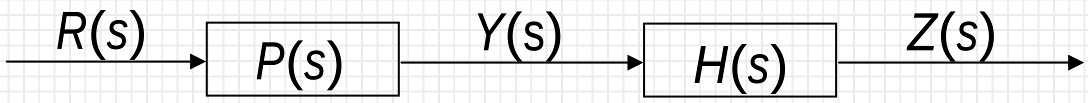
We consider a generic open-loop process as shown below.



A typical design objective is to have the output follow to the input in spite of disturbances and plant model uncertainty.

1.1. Plant Inversion

In a perfect open-loop process without disturbances:

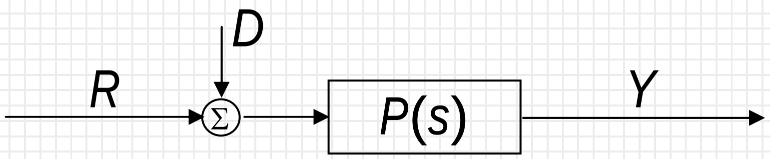


- If P and H are known and fixed and you can measure Z, what should R be to achieve $Y = R^*$?
- Use plant inversion

$R =$ ______________________________

1.2. Feedforward

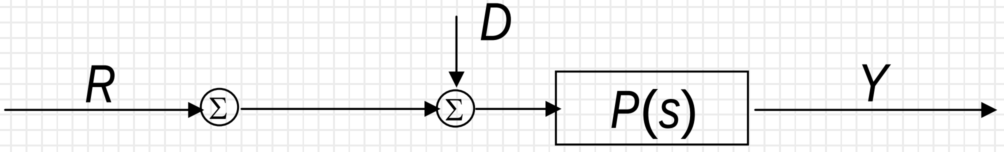
In a perfect open-loop process with disturbances:



- If P is known and fixed, D is known (or measurable), would $R = P^{-1}R^{*}$ achieve $Y = R^{*}$?

- To correctly account for the disturbance:

  - First introduce *feedforward*

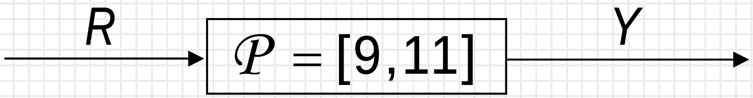


  - then *plant inversion*

$R = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

1.3. Plant Uncertainty

Next consider a more *realistic* open-loop process shown below

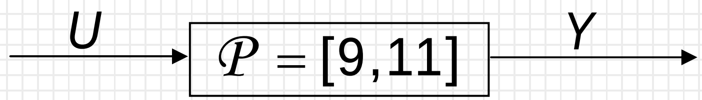


with the spec: $Y = R^* \pm 1\% R^*$ $(= R^*$ at nominal $P_0 = 10)$.

- Solutions:
  - Redesign P?
  - Plant inversion? Which P to use?

Introduce a 2 DOF structure to reduce variations:

- Step 1: introduce *feedback* to reduce uncertainty



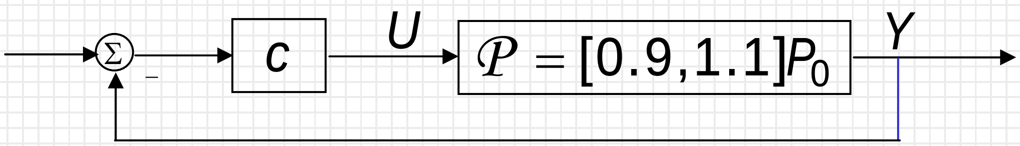
We want variations in Y to be less than 1% of nominal value where

$$Y = \frac{cP}{1+cP}R.$$

At $R = R^*$

$$Y = \begin{Bmatrix} 0.9900R^*, & P = 9 \\ 0.9910R^*, & P = 10 \\ 0.9918R^*, & P = 11 \end{Bmatrix}, \quad \text{so } Y \approx R^* \pm 1\% R^*$$

- Step 2: add a *prefilter* to shape input (2 DOF structure)



- Finally, let $R = R^*$

$$Y = \frac{cPf}{1+cP}R^* = \begin{cases} 0.9990R^*, & P = 9 \\ 1.0000R^*, & P = 10, \\ 1.0008R^*, & P = 11 \end{cases} \quad = R^* \pm 0.001\% R^*$$

1.4. Sensitivity Function Interpretation

- Let

$$T = \frac{cP}{1+cP} \quad \text{and} \quad L = cP$$

$$Y = R^* \pm 1\% R^* \quad \Rightarrow$$

- Define $S_L^T \equiv \lim_{dL \to 0} \frac{dT / T_{max}}{dL / L_{max}} = \frac{dT}{dL} \frac{L_{\max}}{T_{\max}} = \frac{dT}{dL} \frac{L_{\min} + dL}{T_{\min} + dT}$

- Taking limits

- We need

$$S_L^T = \frac{(T_{\max} - T_{\min}) / T_{\max}}{(L_{\max} - L_{\min}) / L_{\max}} = \frac{(T_{\max} - T_{\min}) / T_{\max}}{(cP_{\max} - cP_{\min}) / cP_{\max}}$$

- Compute

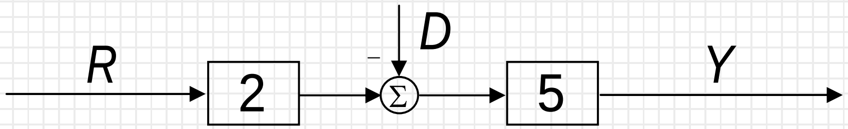
$$Y = \frac{cPf}{1+cP}R^* = \begin{cases} 0.9891R^*, & P = 9 \\ 1.0000R^*, & P = 10, \\ 1.0091R^*, & P = 11 \end{cases} \quad R^* \pm 1.09\% R^*.$$

- Who not 1% precisely?

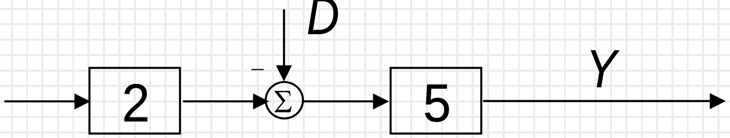
- Prefilter f did not affect sensitivity.

1.5. Disturbance Attenuation

Consider a generic speed control problem shown below. Each 1° road grade causes 5 k/h speed change. Each 1° accelerator change causes 10 k/h change. Design a closed-loop system such that 1° road grade causes only 0.1 k/h speed change.



- Set up a 2DOF structure

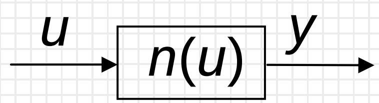


- Computations

$$Y = \frac{10cf}{1+10c}R - \frac{5}{1+10c}D = \underbrace{10R - 0.1D}_{\text{spec}}$$

1.6. Plant Nonlinearities

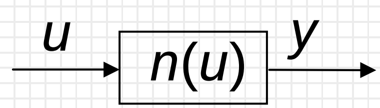
Consider a generic nonlinear plant shown below where



$$n(\mathrm{u}) = u + 5u^3$$

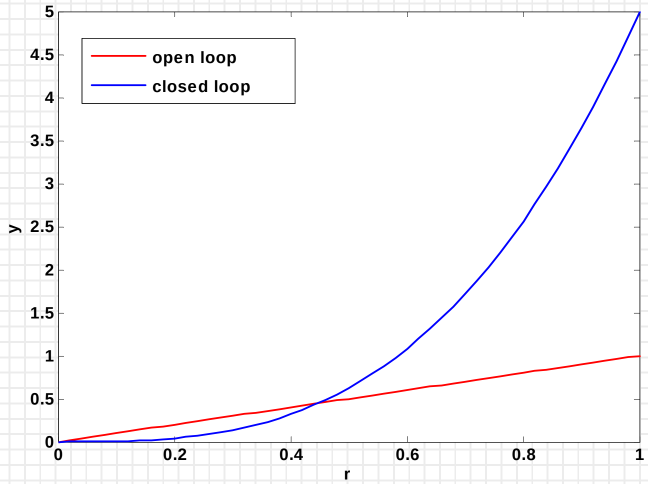
<u>Spec</u>: design the input u such that $y = r \pm \varepsilon$.

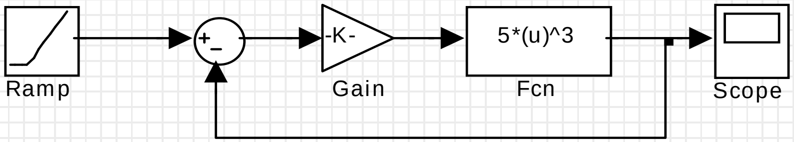
<u>Solution</u>: Introduce a feedback structure



- Methods: cut-and try, equivalent disturbance, cancellation (feedback linearization).

• Cut-and try:





• Equivalent disturbance

• Feedback linearization

1.7. Summary

- Why feedback?